

## 1.02 Calculators and Scientific Notation

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**ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE**

At the time of this writing (1994), there are a variety of different calculators available at a variety of prices. If you are planning to take higher mathematics courses, or if you wish to use a state-of-the-art calculator, you will probably want a "graphics" calculator. A graphics calculator will certainly be useful in this course, but not necessary. An "ordinary" scientific calculator will be sufficient. For this course, several features will be necessary. **First**, you will need **logarithmic functions "log x" and "ln x."** [Note: "ln x" is pronounced "el en of x."] (If your calculator has one of these, it probably has the other as well!) If your calculator has the **"ln" function**, then it also has the **"e<sup>x</sup> function"**. [Note: "e" is a real number whose value is approximately 2.718. This number is the base of the **natural logarithms** system, which will be explained in Chapter 4.]

**Second**, you will need a **"power" function**. Look for the **"x<sup>y</sup>"** or the **"y<sup>x</sup>" function** on the calculator. The opposite of **raising X or Y to a power** is **taking a root of X or Y**. This is the **root function**, which is indicated on the calculator by the keys

$X^{\frac{1}{y}}$  ( $\sqrt[y]{X}$ ) or  $Y^{\frac{1}{x}}$  ( $\sqrt[x]{Y}$ ). For convenience, your calculator probably has the **square (X<sup>2</sup>)** and **square root ( $\sqrt{X}$ ) functions**. It will also be helpful (but not necessary) for your calculator to have **parentheses "(" and ")" keys**, **memory function(s)**, and **reciprocal function (1/X)**. In this section, you will find some explanation, examples, and exercises which will supplement your calculator instructions and help you practice with your calculator. Please remember that each brand and model of calculator is different, and it is important that you learn to work with your calculator, and that you practice with it!!! (Also, have an extra battery on test day!)

Use your calculator to compute the following exercises:

1.  $6,000 \times 7,000 =$  \_\_\_\_\_
2.  $60,000 \times 70,000 =$  \_\_\_\_\_
3.  $600,000 \times 700,000 =$  \_\_\_\_\_
4.  $6,000,000 \times 7,000,000 =$  \_\_\_\_\_

At some point in these calculations, depending upon your calculator, the answer began with a reasonable **42,000,000** for #1, and then "degenerated" to what on your calculator probably looks like **4.2 13** or **4.2 E 13** or **4.2<sup>13</sup>** as the answer to #4. What happened? Is the calculator correct? Actually, the problem is that the calculator can only display so many digits--some calculators can only display eight digits, others can display ten digits--on the screen. It is therefore necessary for **the calculator to convert the answer to scientific notation!!** The answer to #4 is actually **4.2 x 10<sup>13</sup>**.

**PRINCIPLE: When learning a new skill,  
begin with examples to which  
you already know the answer!**

When raising to a power, usually you begin by entering the **base number**, then "**X**", then the **power**, then "**enter**" or "**=**". When squaring, it may be faster to use the **X<sup>2</sup>** function. [Note: the Texas Instruments TI-85 (and probably others) have a "**^**" key to signify raising to a power.]

- |                      |                        |
|----------------------|------------------------|
| 5a) $4^2 =$ _____    | b) $4567^2 =$ _____    |
| 6a) $4^3 =$ _____    | b) $4567^3 =$ _____    |
| 7a) $2^5 =$ _____    | b) $4567^5 =$ _____    |
| 8a) $2^{10} =$ _____ | b) $4567^{10} =$ _____ |

You probably noticed that when the calculator converts an answer to scientific notation, the answer is frequently rounded off. Or, when calculating an answer that does not come out even, the calculator may give the answer with seven or more digits of accuracy. In such cases, how many digits of accuracy should you give in your answer? How should it be rounded off? There are different ways to define and justify rounding procedures. The easiest way is to round to a given number of decimal places: two decimal places (nearest hundredth), or three decimal places (nearest thousandth), etc. If the digit "beyond" the round-off digit is greater than or equal to 5, round up; if the digit "beyond" the round-off digit is less than 5, then round down. **Whatever you do, NEVER USE A ROUNDED ANSWER TO CALCULATE ANOTHER ANSWER!!** Depending upon the numbers involved in the calculation, the round-off error that results from using a rounded-off value could be incredible!!

**PRINCIPLE: Never use a rounded-off answer to calculate another answer! If possible use the full accuracy of the calculator.**

#### **GRAPHICS CALCULATOR NOTE**

When taking a square root, cube root, etc., you usually enter the operation to be performed (square root, cube root, etc.) first and then you enter the radicand (the number inside the radical).

#### **NON-GRAPHICS CALCULATOR NOTE**

For most non-graphics calculators, you must enter the radicand (the number inside the radical) first, then you enter the operation to be performed (square root, cube root, etc.).

Compute the square roots, first by using the square root function, and then by raising to the one-half power. Round answers to the nearest hundredth.

9.  $\sqrt{64}$  = \_\_\_\_\_  $64^{.5}$  = \_\_\_\_\_
10.  $\sqrt{640}$  = \_\_\_\_\_  $640^{.5}$  = \_\_\_\_\_
11.  $\sqrt{1,000,000}$  = \_\_\_\_\_  $1,000,000^{.5}$  = \_\_\_\_\_
12.  $\sqrt{10,000,000}$  = \_\_\_\_\_  $10,000,000^{.5}$  = \_\_\_\_\_

When taking a cube root, fourth root, or in general the  $n$ th root of a number, the procedure will vary from calculator to calculator. Be sure to see the notes on the previous page. If your calculator has the  $Y^X$  or  $X^Y$  function, then it probably has a function  $\sqrt[n]{\phantom{x}}$  or  $64^{\frac{1}{2}}$  nearby. The  $\sqrt[n]{\phantom{x}}$  or  $64^{\frac{1}{2}}$  is probably printed above the  $Y^X$  or  $X^Y$  key or else on the same key but in a different color. This indicates that you must first press the "2nd" (or perhaps "alternate" or "inverse") function, and then press this key in order to activate this function. Next, press 3 for cube root, 4 for 4th root, 5 for 5th root, etc., and finally, press "enter" to obtain the final answer. When calculating with the exponent instead of the radical, remember to use the full accuracy of the calculator (not a rounded-off value!) for the exponent.

**Special TI-85 Calculator Note:**  
 To access the  $\sqrt[n]{\phantom{x}}$  function on the TI-85, use the following keystrokes: 2nd, Math, F5, More, [index of radical], F4, [radicand], enter. For example, to compute the  $\sqrt[5]{32}$ , use these steps: 2nd, Math, F5, More, 5, F4, 32, enter.

13.  $\sqrt[4]{16}$  = \_\_\_\_\_  $16^{.25}$  = \_\_\_\_\_
14.  $\sqrt[4]{81}$  = \_\_\_\_\_ \_\_\_\_\_ = \_\_\_\_\_

15.  $\sqrt[5]{32} = \underline{\hspace{2cm}}$        $32 \cdot 2 = \underline{\hspace{2cm}}$

16.  $\sqrt[3]{64} = \underline{\hspace{2cm}}$        $64 \cdot 33 = \underline{\hspace{2cm}}$

$64 \cdot 333 = \underline{\hspace{2cm}}$        $64 \cdot 333333333 = \underline{\hspace{2cm}}$

17.  $\sqrt[3]{5678} = \underline{\hspace{2cm}}$        $5678^{\frac{1}{3}} = \underline{\hspace{2cm}}$

18.  $\sqrt[4]{5678} = \underline{\hspace{2cm}}$        $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

19.  $\sqrt[5]{1,000,000} = \underline{\hspace{2cm}}$        $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

20.  $\sqrt[4]{100,000,000} = \underline{\hspace{2cm}}$        $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Consider now the problem of combined operations, multiplying, dividing, and raising to powers. For example, consider the problem

$\frac{7.2 \cdot 6.3}{3.5 \cdot 8.1}$ . According to the order of operations agreement, it

would be correct to begin with the **product of the numerator factors**, then " $\div$ ", then **open parentheses**, then the **product of the denominator factors**, **close parentheses**, and " $=$ " or "**enter**." You may wish to note that if you omit the "**close parentheses**," the result is the same. Try this problem for yourself:

21.  $\frac{7.2 \cdot 6.3}{3.5 \cdot 8.1} = \underline{\hspace{2cm}}$ .

Another approach to this same problem, especially if you do not have the **parentheses function**, is to realize that all numerator factors represent multiplication, and all denominator factors represents division. Therefore, you may begin with the **product of the numerator factors**, then "**÷**" the **first denominator factor**, then "**÷**" the **second denominator factor**, then "**=**" or "**enter**." Try it this way, and see if the result is the same:

21.  $\frac{7.2 \cdot 6.3}{3.5 \cdot 8.1} = \underline{\hspace{2cm}}$ .

22.  $\frac{72,000 \cdot 0.0063}{0.035 \cdot 81} = \underline{\hspace{2cm}}$ . 23.  $\frac{0.72 \cdot 6,300,000}{350 \cdot 810} = \underline{\hspace{2cm}}$ .

24.  $\frac{0.000984 \cdot 1,940}{8,560,000 \cdot 0.76} = \underline{\hspace{2cm}}$ . 25.  $\frac{0.0096 \cdot 0.0000042}{8350 \cdot 0.081} = \underline{\hspace{2cm}}$ .

26.  $\frac{72,000^2 \cdot 0.0063}{0.035 \cdot 81^2} = \underline{\hspace{2cm}}$ .

27.  $\frac{0.72 \cdot 6,300,000^2}{350^2 \cdot 810} = \underline{\hspace{2cm}}$ .

28.  $\frac{0.000984^2 \cdot 1,940^2}{8,560,000^2 \cdot 0.76^2} = \underline{\hspace{2cm}}$ . [Hint: See #24.]

29.  $\frac{0.0096^2 \cdot 0.0000042^2}{8350^2 \cdot 0.081^2} = \underline{\hspace{2cm}}$ .

It is frequently necessary to use **scientific notation**, and to be able to enter scientific notation into the calculator. Again this depends upon the calculator, since some calculators perform calculations automatically according to the order of operations, while some calculators perform the calculations in the order that they are input. To illustrate this, please use your own calculator to enter  $7.2 \times 10^{12}$  by entering the following keystrokes:

7.2, times, 10,  $X^Y$  or  $Y^X$ , 12, = or "enter" \_\_\_\_\_  
(Write your answer!)

Of course, you should have an answer of  $7.2 \times 10^{12}$ . If you did, then your calculator performed according to the order of operations agreement. However, some calculators get the answer  $1.9408 \times 10^{22}$ ! This is obviously **wrong**, but how did they get it? Answer: the calculator performed the operations in the order they were given: multiplied 7.2 times 10, then raised 72 to the 12th power. You need to know how your own calculator operates. This is why the following principle is so important:

**Principle:** You must practice with and become familiar with your own calculator! During an examination is not a good time to do this!!

Some calculators have the **EE (or EXP) function**, which makes **scientific notation** much easier to enter. If you have an **EE or EXP key** on your calculator, to enter  $7.2 \times 10^{12}$ , simply enter 7.2, **EE (or EXP)**, 12, **enter**. It doesn't get any easier than that! There is one more function or skill you will need for scientific notation. What about **negative exponents**, such as  $7.2 \times 10^{-12}$ ? This is done using what is called the **"+/-" key**. Whatever you did to enter  $7.2 \times 10^{12}$ , try it now by touching the **"+/-" key** just before you enter 12. Try it to be sure: 7.2, **EE (EXP)**, **"+/-"**, 12, **enter**: \_\_\_\_\_  
(Your answer)

(One more for practice--enter into your calculator:  $8.6 \times 10^{39}$ .)

Frequently, there will be combined operations, as the following example:  $\frac{7.2 \times 10^{12} \cdot 6.3 \times 10^{-8}}{3.5 \times 10^{-4} \cdot 8.1 \times 10^{14}}$

It will be helpful to work up to this example "one step at a time!"

30.  $7.2 \times 10^{12} \cdot 6.3 \times 10^{-8} =$

31.  $3.5 \times 10^{-4} \cdot 8.1 \times 10^{14} =$

32.  $\frac{7.2 \times 10^{12}}{3.5 \times 10^{-4}} =$

33.  $\frac{6.3 \times 10^{-8}}{8.1 \times 10^{14}} =$

34.  $\frac{7.2 \times 10^{12}}{8.1 \times 10^{-14}} =$

35.  $\frac{7.2 \times 10^{12} \cdot 6.3 \times 10^{-8}}{3.5 \times 10^{-4} \cdot 8.1 \times 10^{14}} =$

36.  $\frac{6.3 \times 10^{23} \cdot 9.5 \times 10^{-4}}{7.5 \times 10^{-12} \cdot 4.3 \times 10^3} =$

37.  $\frac{72,000^2}{\sqrt{0.035}} =$

38.  $\frac{\sqrt[3]{0.72} \cdot 6,300,000}{\sqrt{350}} =$

$$39. \frac{1,940^4}{8,560,000 \cdot 0.067^3} =$$

$$40. \frac{0.0096^2 \cdot \sqrt{0.00042}}{8350^3} =$$

$$41. (7.2 \cdot 10^{12})^2 \cdot (6.3 \cdot 10^5)^3 =$$

$$42. (9.4 \cdot 10^{-12})^3 \cdot (8.3 \cdot 10^5)^2 =$$

$$43. \frac{\sqrt{7.2 \times 10^{12}}}{3.5 \times 10^{-4}} =$$

$$44. \frac{(6.3 \times 10^{-8})^3}{(8.1 \times 10^{14})^2} =$$

$$45. \frac{(7.2 \times 10^{12})^3}{(8.1 \times 10^{-14})^2} =$$

$$46. \sqrt{\frac{7.2 \times 10^{12} \cdot 6.3 \times 10^{-8}}{3.5 \times 10^{-4} \cdot 8.1 \times 10^{14}}} =$$

[Hint: See #35.]

## ANSWERS 1.02

- p. 15-22: 1.  $4.2 \times 10^7$  2.  $4.2 \times 10^9$  3.  $4.2 \times 10^{11}$  4.  $4.2 \times 10^{13}$   
 5a) 16 6)  $2.08 \times 10^7$  6a) 64 7)  $9.53 \times 10^{10}$  7a) 32  
 7b)  $1.99 \times 10^8$  8a) 1024 8b)  $3.95 \times 10^{36}$  9. 8 10. 25.30  
 11. 1080 12. 3162.28 13. 2 14. 3 15. 2 16. 4,  
 3.94, 3.99, 4.00 17. 17.84 18. 8.68 19. 15.85  
 20. 100 21. 1.60 22. 160 23. 16 24.  $2.93 \times 10^{-7}$   
 25.  $5.96 \times 10^{-4}$  26. 142222.22 27. 288000  
 28.  $8.61 \times 10^{-14}$  29.  $3.55 \times 10^{-21}$  30.  $4.54 \times 10^5$   
 31.  $2.84 \times 10^{11}$  32.  $2.06 \times 10^{16}$  33.  $7.78 \times 10^{-23}$   
 34.  $8.89 \times 10^{25}$  35.  $1.6 \times 10^{-6}$  36.  $1.86 \times 10^{28}$   
 37.  $2.77 \times 10^{10}$  38.  $3.02 \times 10^5$  39.  $5.50 \times 10^9$   
 40.  $3.24 \times 10^{-18}$  41.  $1.30 \times 10^{43}$  42.  $5.72 \times 10^{-22}$   
 43.  $7.67 \times 10^9$  44.  $3.81 \times 10^{-52}$  45.  $5.69 \times 10^{64}$   
 46.  $1.26 \times 10^{-3}$

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