3.03 *Polynomial Functions*

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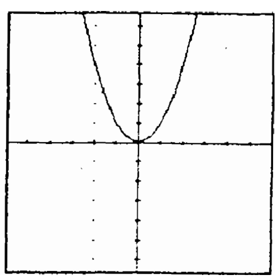
ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

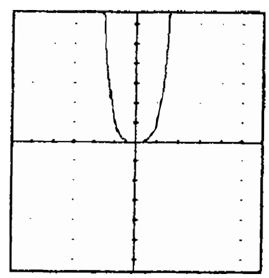
In a previous section, you have graphed functions of the form $\mathbf{Y = aX^2}$, and you recall that the graph is a **parabola** opening **upward** for a>0, and opening **downward** for a<0. In this section, functions such as $\mathbf{Y = aX^3}$ and $\mathbf{Y = aX^4}$ will be graphed. These functions, in general form $\mathbf{Y = a_nX^n + a_{n-1}X^{n-1} + a_{n-2}X^{n-2} + \ldots a_0}$, are known as **polynomial functions**.

Consider the following series of graphs obtained by just plotting points:

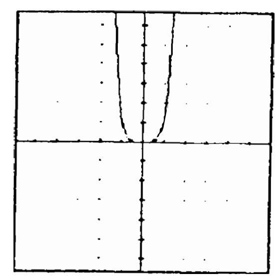
1. $\mathbf{Y = X^2}$



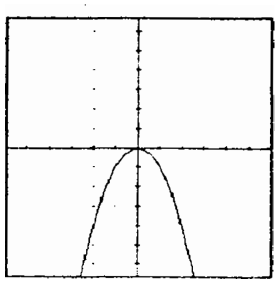
2. $\mathbf{Y = X^4}$



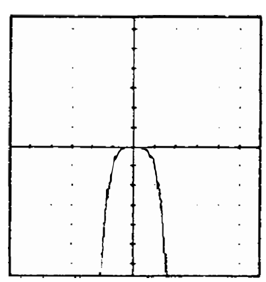
3. $\mathbf{Y = X^6}$



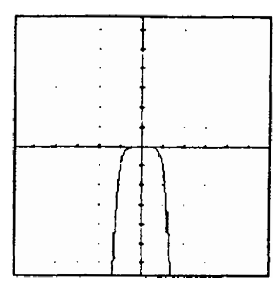
4. $\mathbf{Y = -X^2}$



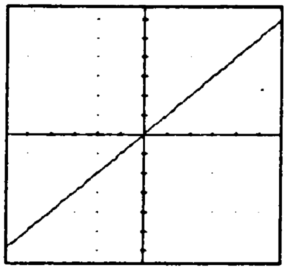
5. $\mathbf{Y = -X^4}$



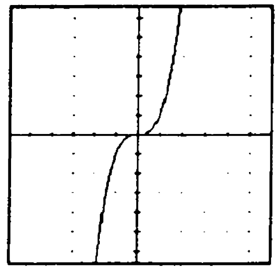
6. $\mathbf{Y = -X^6}$



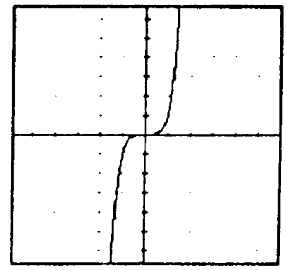
7. $Y = X$



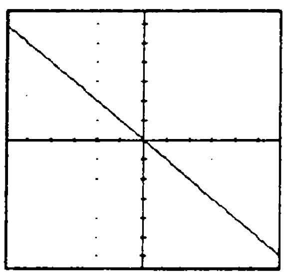
8. $Y = X^3$



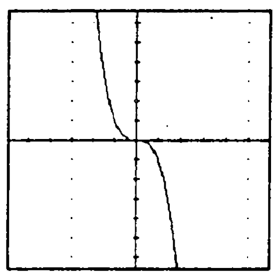
9. $Y = X^5$



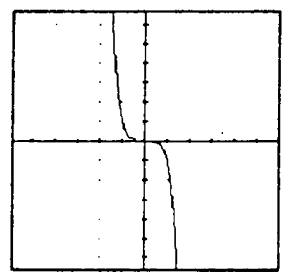
10. $Y = -X$



11. $Y = -X^3$



12. $Y = -X^5$



Graphics by Paul Joachim.

Notice that the graphs of each of the equations $\mathbf{Y = aX^n}$ above passes through the pont **(0,0)**, and **(0,0)** is the only X or Y-intercept. Notice also that in #1-3, where $Y=X^{(\text{even})}$, the graph opens "up" on both sides, while in #4-6, where $Y=-X^{(\text{even})}$, the graph opens "down" on both sides. In #7-9, $Y=X^{(\text{odd})}$, the graph opens "up" on the right and "down" on the left, while in #10-12, $Y=-X^{(\text{odd})}$, this is reversed, opening "down" on the right and "up" on the left side.

When there are additional terms in the polynomial function, **[for example, $Y = X^3 - 2X^2 - X + 2$]**, it is the degree (highest power of X) and the sign of the leading coefficient that determine the "overall shape" of the graph. By factoring, you can find the X-intercepts of the function, and these determine the shape of the "interior" of the graph. **The X-intercepts are called the zeros of the function.**

EXERCISES:

1. **Graph $Y = X^3 - 2X^2 - X + 2$ by finding X and Y-intercepts.**

SOLUTION: Y-intercept (X=0) $Y = 2 \Rightarrow (0,2)$

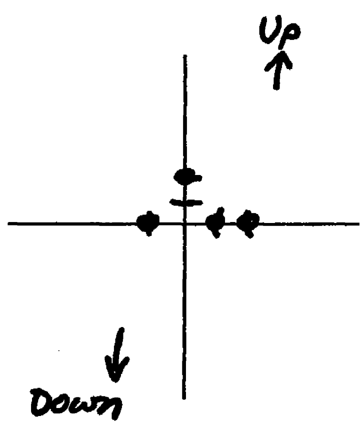
X-intercepts (Y=0) $0 = X^3 - 2X^2 - X + 2$

Solve by factoring: $0 = X^2(X-2) - 1(X-2)$

Degree = 3 $(X-2)(X^2 - 1) = 0$

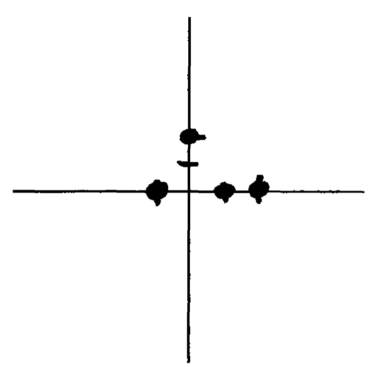
Leading Coef = + $(X-2)(X-1)(X+1) = 0$

$X=2$ $X=1$ $X=-1 \Rightarrow (2,0)$ $(1,0)$ $(-1,0)$



Given these intercepts, recall that the overall shape is like $Y=X^3$ (up on right, down on left!).

Now, you sketch the final graph in the space provided below:



2. Graph $Y = X^4 - 5X^2 + 4$ by finding X and Y-intercepts.

SOLUTION: Y-intercept (X=0) $Y = 4 \Rightarrow (0,4)$

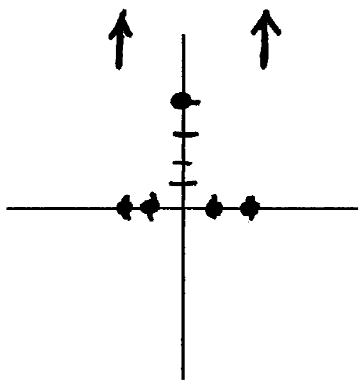
X-intercepts (Y=0) $0 = X^4 - 5X^2 + 4$

Solve by factoring: $0 = (X^2-4)(X^2-1)$

Degree = 4 $(X-2)(X+2)(X-1)(X+1) = 0$

Leading Coef = + $X=2 \quad X=-2 \quad X=1 \quad X=-1$

$(2,0) \quad (-2,0) \quad (1,0) \quad (-1,0)$



The overall shape is like $Y=X^4$ -- opens up on both sides! Now, complete the graph at the left with a smooth curve.

So far, the method of graphing polynomial functions has included finding the Y-intercept, finding the zeros (i.e., the X-intercepts), and identifying the "overall shape" of the graph. However, in addition to finding the zeros of the polynomial functions, it will be helpful to identify the multiplicities of each zero and to understand how these multiplicities affect the graph.

To understand the effect of multiple zeros on the graph, return to the first twelve graphs of this section. The polynomial functions $Y=X$, $Y=X^3$, and $Y=X^5$ all have zeros at $X=0$, with multiplicities 1, 3, and 5 respectively. These graphs, with **odd** multiplicities, all pass **through** the zero.

The polynomial functions $Y=X^2$, $Y=X^4$, and $Y=X^6$ all have zeros at $X=0$, with multiplicities 2, 4, and 6 respectively. These graphs, with **even** multiplicities, do not pass through the zero, but rather they **touch** the X-axis and **bounce** back.

DEGREE OF POLYNOMIAL

ODD: **Coefficient positive--Up on right, down on left.**
Coefficient negative--Down on right, up on left.

EVEN: **Coefficient positive--Up on both sides.**
Coefficient negative--Down on both sides.

MULTIPLICITY OF ROOTS

ODD: **Passes through the X-axis.**

EVEN: **Bounces back from the X-axis.**

A graphics calculator or techniques from calculus may be used to determine where the graph "turns around" (that is, the relative maximum and minimum points.) For now, it is not necessary to determine these points.

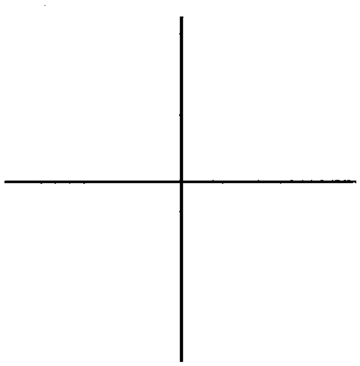
In 3-40, graph the polynomial functions by finding the Y-intercept and zeros. (NOTE: #3-6 are parabolas, but for now, vertices are not necessary!)

3. $Y = X^2 - 5X + 6$

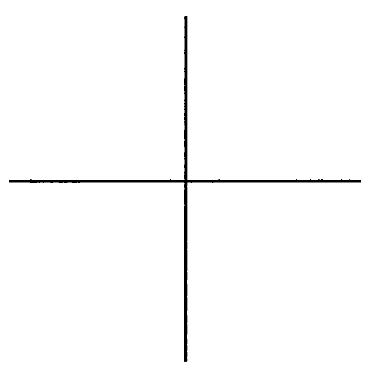
Y-int: (X=0) Y = ______

Zeros: (Y=0) (X) (X)=0

X=_____; X=______



4. $Y = X^2 - X - 6$



5. $Y = -X^2 - X + 6$

6. $Y = -X^2 + 2X + 8$

7. $Y = (X - 1)(X + 3)(X + 1)$

Degree of polynomial = 3

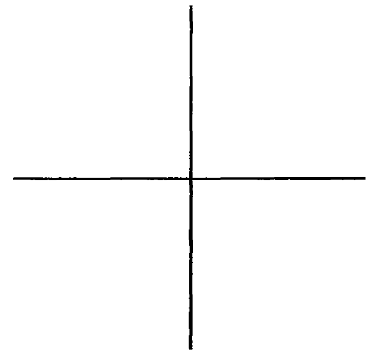
Leading coefficient = +

Zeros: ______, ______, ______

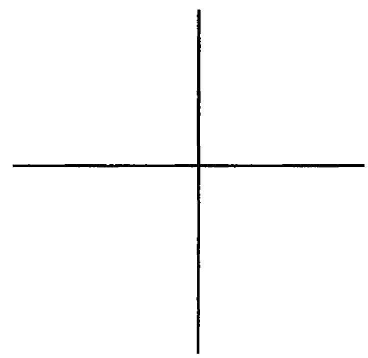
Y-intercept: (X=0) ________.

8. $Y = (X + 1)(X - 3)(X - 1)$

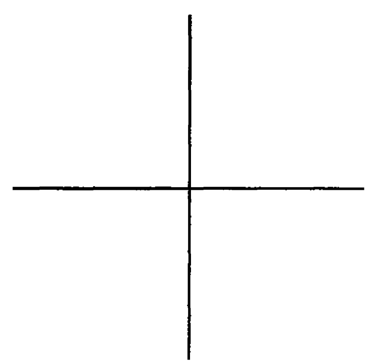
9. $Y = X(X - 1)(X - 2)(X + 3)$



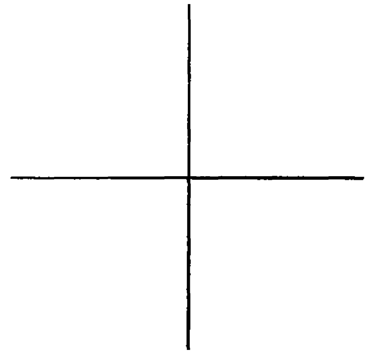
10. $Y = X(X - 1)(X + 2)(X - 3)$



11. $Y = (X - 1)(X - 3)(X + 1)(X + 2)$

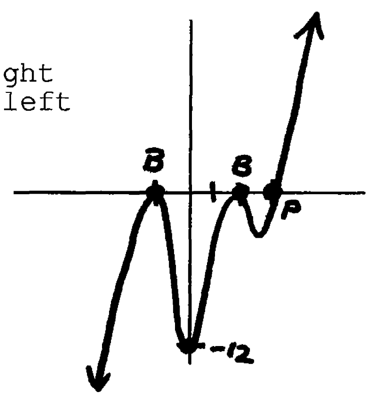


12. **$Y = (X - 1)(X - 2)(X + 1)(X + 3)$**



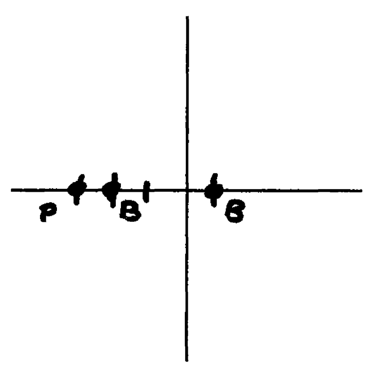
13. **$Y = (X - 2)^2(X + 1)^4(X - 3)$**

Degree of polynomial = 7 Odd--Up on right
Leading coefficient = + Down on left
Y-intercept: (X=0) $Y=(-2)^2(1)^4(-3)$
$= \underline{-12}$

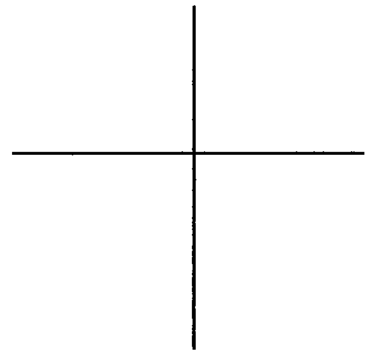


Zeros	Multiplicity
2	2 (Even--Bounces!)
-1	4 (Even--Bounces!)
3	1 (Odd--Passes thru!)

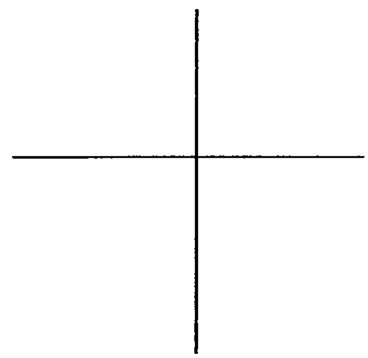
14. **$Y = (X + 2)^2(X - 1)^4(X + 3)$**



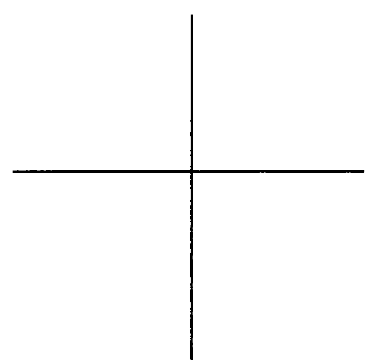
15. $Y = (X - 3)(X - 1)^2(X + 2)(X + 1)$



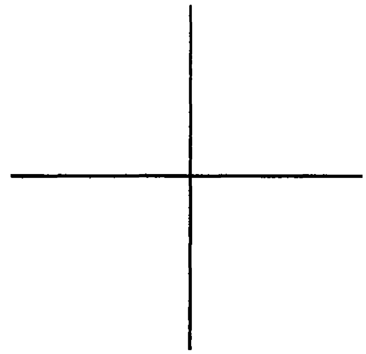
16. $Y = (X - 3)(X - 1)^2(X - 2)(X + 2)$



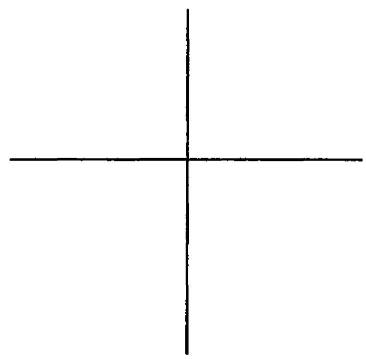
17. $Y = -(X - 3)(X - 1)^3(X + 2)^2$



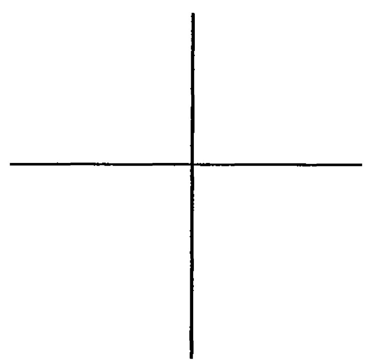
18. $Y = -(X + 3)(X + 1)^3(X - 2)^2$



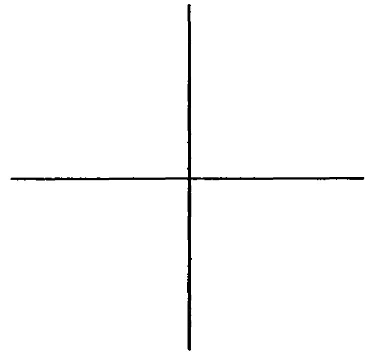
19. $Y = X(X - 1)^2(X + 2)^2(X - 3)^2$



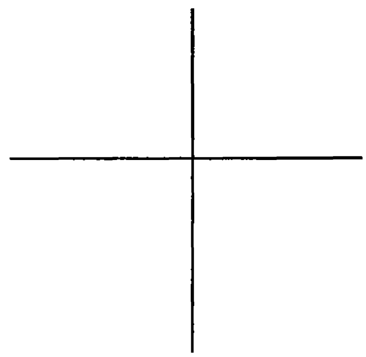
20. $Y = X(X + 1)^2(X - 2)^2(X + 3)^2$



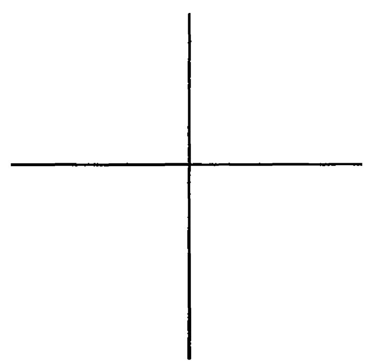
21. $Y = X^2(X - 2)^2(X + 3)(X + 1)^3$



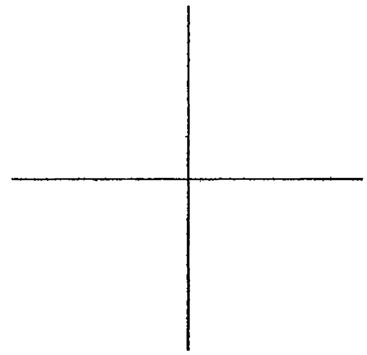
22. $Y = X^2(X + 2)^2(X - 3)(X - 1)^3$



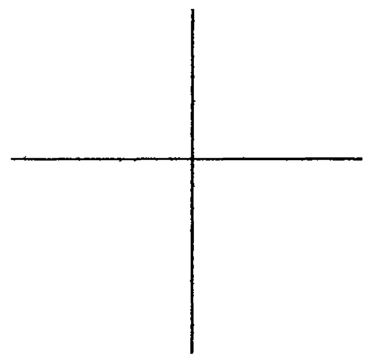
23. $Y = X^3(X + 2)^4(X - 2)$



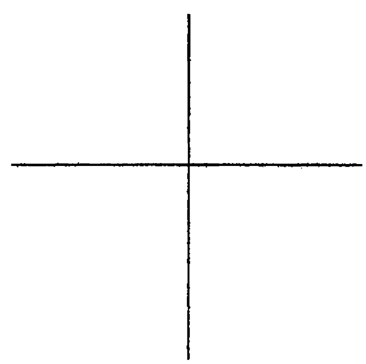
24. $Y = X^3(X - 2)^4(X + 2)^2$



25. $Y = -X^2(X - 1)^2(X + 3)^2(X - 4)$



26. $Y = -X^2(X + 1)^2(X - 3)(X + 4)^2$



27. $Y = X^4 - 10X^2 + 9$

$= (\quad\quad)(\quad\quad)$

$= (\quad\quad)(\quad\quad)(\quad\quad)(\quad\quad)$

Degree = ____________

Sign of coeff = _____

Y-intercept = ______

Zeros: ______________

28. $Y = X^4 - 13X^2 + 36$

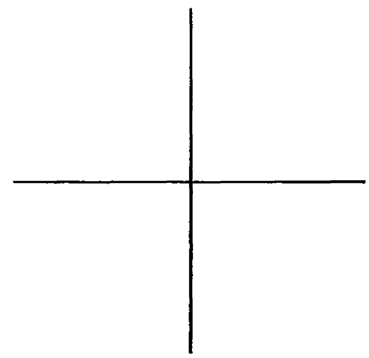
29. $Y = -X^4 + 10X^2 - 9$

$= -(\quad\quad\quad\quad)$

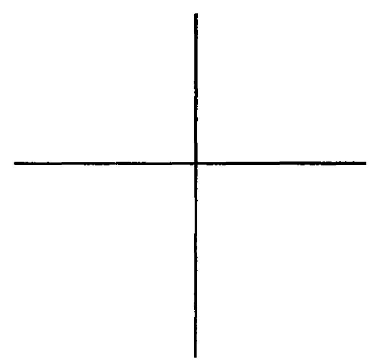
$= -(\quad\quad)(\quad\quad)$

$= -(\quad)(\quad)(\quad)(\quad)$

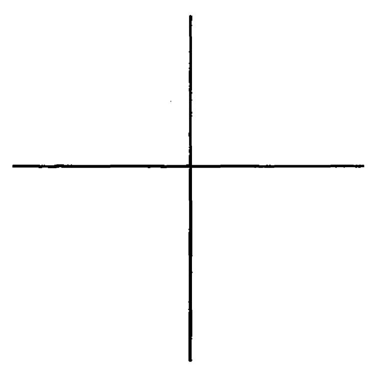
30. $Y = -X^4 + 13X^2 - 36$



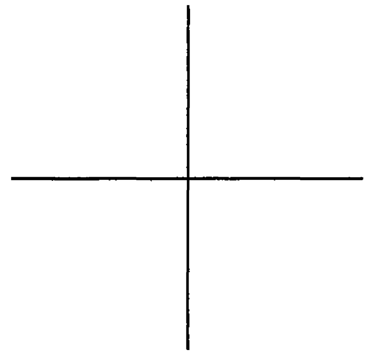
31. $Y = X^3 - X^2 - 9X + 9$



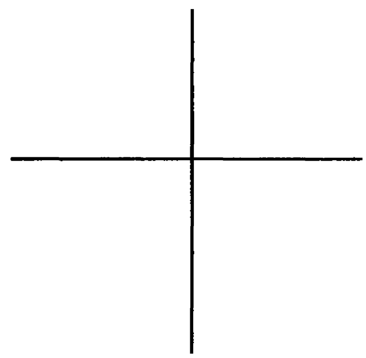
32. $Y = X^3 + 4X^2 - X - 4$



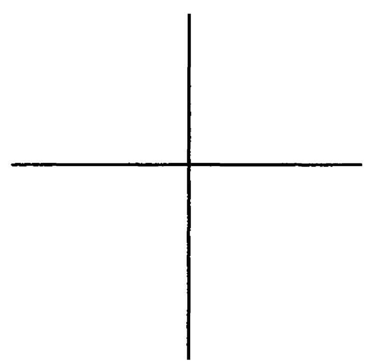
33. $Y = X^3 - 3X^2 - 9X + 27$



34. $Y = X^3 + 2X^2 - 4X - 8$



35. $Y = X^4 - 8X^2 + 16$

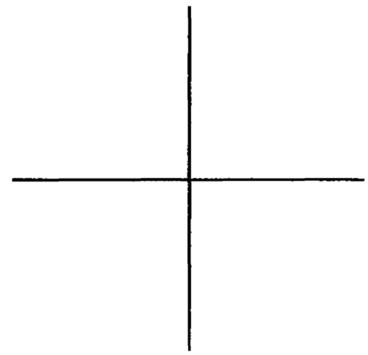


36. $Y = X^4 - 18X^2 + 81$

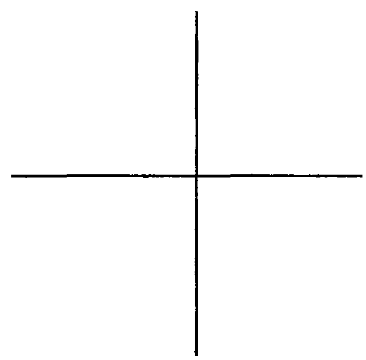
37. $Y = X^6 - 13X^4 + 36X^2$

38. $Y = X^7 - 13X^5 + 36X^3$

39. $Y = X^5 + 2X^4 - 4X^3 - 8X^2$



40. $Y = X^7 - 8X^5 + 16X^3$



3.03 p.363-377:

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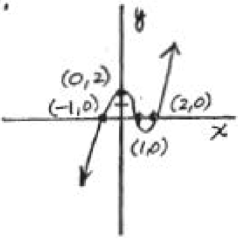

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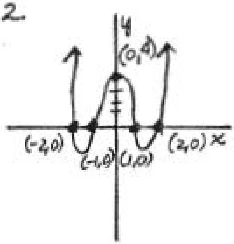

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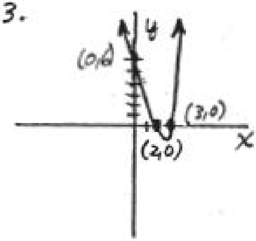

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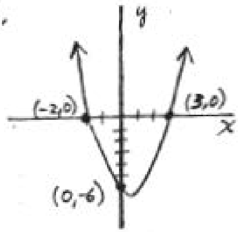

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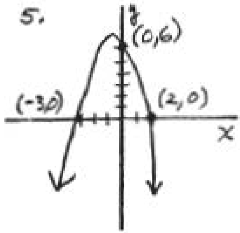

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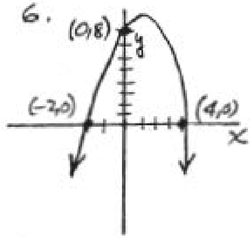

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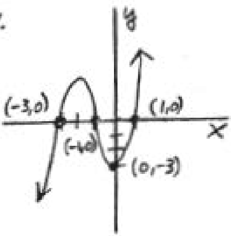

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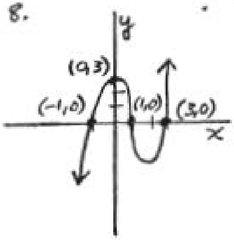

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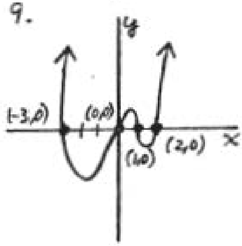
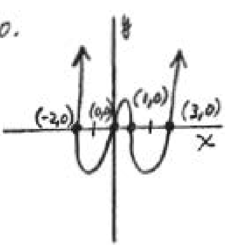


10.



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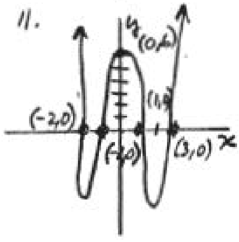

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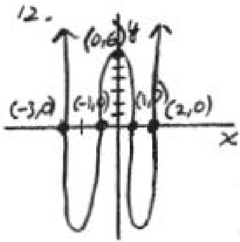

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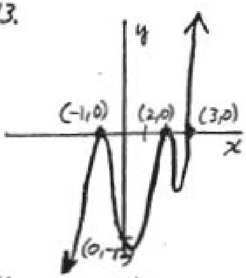

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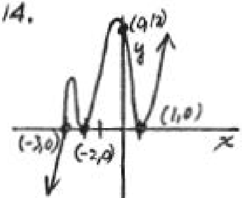

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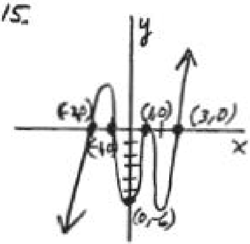

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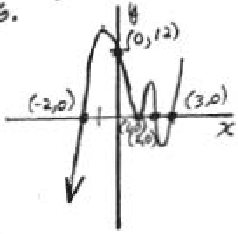

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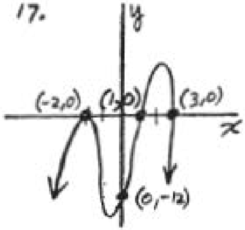

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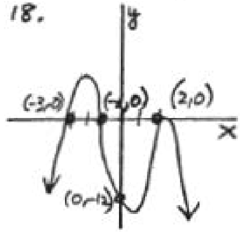

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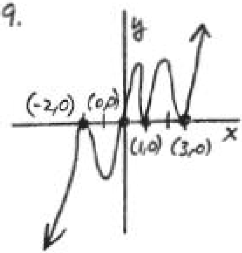

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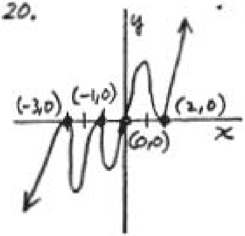

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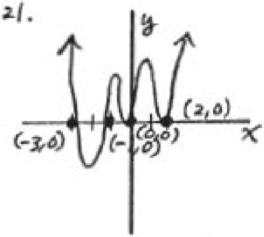

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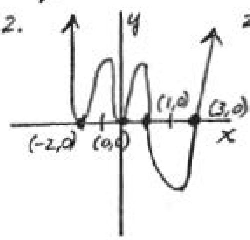

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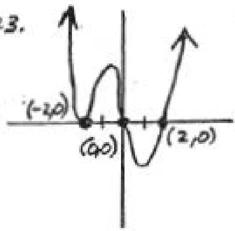

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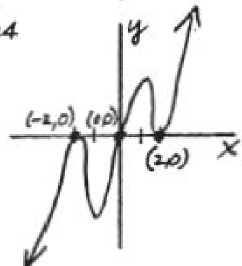

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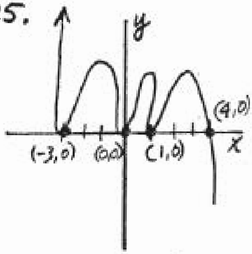

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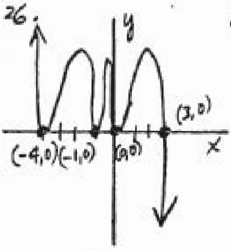

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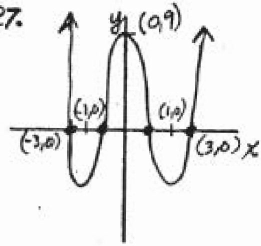

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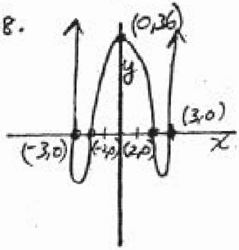

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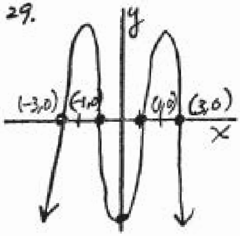

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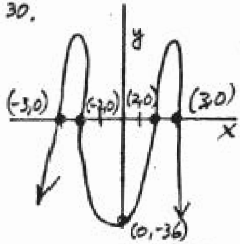

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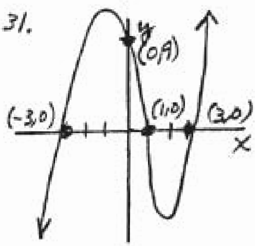

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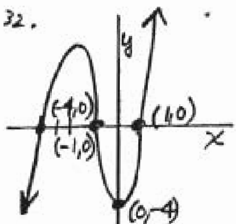

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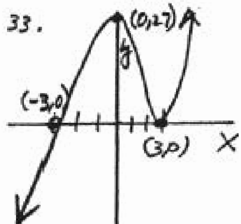

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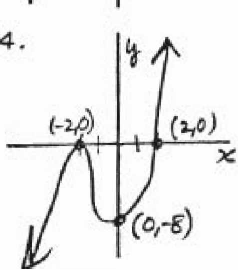

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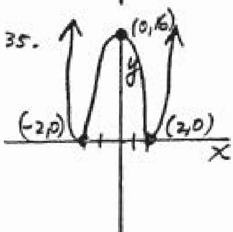

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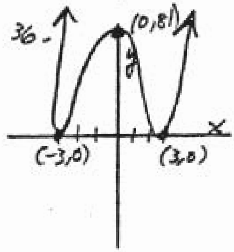

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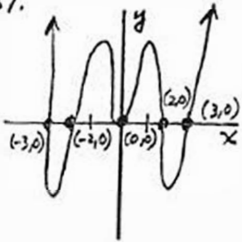

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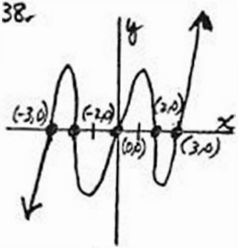

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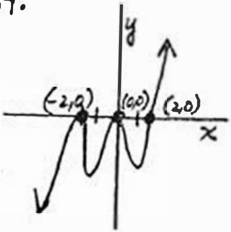

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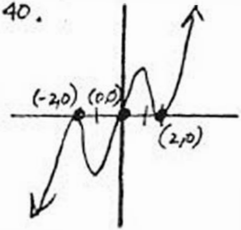

Dr. Robert J. Rapalje

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE