Math in Living C O L O R !!

2.04 Multiplying and Dividing Fractions

Intermediate Algebra: One Step at a Time
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See Section 2.04 with explanations, examples, and exercises, coming soon!

Explanations, examples, and exercises from Basic Algebra, coming soon!

P. 159. #19. $\dfrac{42x^7}{27y^3} \div \dfrac{14x^6}{9y^5}$

Solution: This is a division problem, so you must invert the second fraction and multiply. Several factors will divide out. Notice that these are **FACTORS**, not **TERMS**.

$$\frac{42x^7}{27y^3} \cdot \frac{9y^5}{14x^6}$$

First, 14 divides into 42 and it goes 3 times, and 9 divides into 27, and it goes 3 times.

$$\frac{3\cancel{42}x^7}{3\cancel{27}y^3} \cdot \frac{\cancel{9}y^5}{\cancel{14}x^6}$$

Next, the 3 factors divide out:

$$\frac{x^7}{y^3} \cdot \frac{y^5}{x^6}$$

Now, the x^6 factor divides out with the x^7 leaving an x factor in the numerator, and the y^3 factor divides out with the y^5 leaving a y^2 factor in the numerator. It looks like this:

$$\frac{x\cancel{x^7}}{\cancel{y^3}} \cdot \frac{\cancel{y^5}y^2}{\cancel{x^6}}$$

The **final answer** is $\dfrac{xy^2}{1}$ or xy^2.

P. 160 # 24.

$$\frac{x^2-x}{x^2-x-12} \cdot \frac{x^2-3x-4}{x^2-1}$$

Solution: This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{x(x-1)}{(x-4)(x+3)} \cdot \frac{(x-4)(x+1)}{(x-1)(x+1)}$$

Notice that the (x-1) in the first numerator and the (x-1) in the second denominator divide out.

$$\frac{x\cancel{(x-1)}}{(x-4)(x+3)} \cdot \frac{(x-4)(x+1)}{\cancel{(x-1)}(x+1)}$$

$$\frac{x}{(x-4)(x+3)} \cdot \frac{(x-4)(x+1)}{(x+1)}$$

Also the (x-4) in the first denominator and the second numerator divide out.

$$\frac{x}{\cancel{(x-4)}(x+3)} \cdot \frac{\cancel{(x-4)}(x+1)}{(x+1)}$$

$$\frac{x}{(x+3)} \cdot \frac{(x+1)}{(x+1)}$$

And the (x+1) in the second numerator and denominator divide out.

$$\frac{x}{(x+3)} \cdot \frac{\cancel{(x+1)}}{\cancel{(x+1)}}$$

What is left is the x in the numerator and the (x+3) in the denominator.

Final answer: $\dfrac{x}{(x+3)}$

P. 160 # 25.

$$\frac{x^2-49y^2}{x^2+12xy+35y^2} \cdot \frac{x^2-3xy-10y^2}{x^2-5xy-14y^2}$$

Solution: This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{(x-7y)(x+7y)}{(x+7y)(x+5y)} \cdot \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

Notice that the (x+7y) in the first numerator and the (x+7y) in the first denominator divide out.

$$\frac{(x-7y)\cancel{(x+7y)}}{\cancel{(x+7y)}(x+5y)} \cdot \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

$$\frac{(x-7y)}{(x+5y)} \cdot \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

Also the (x+2y) in the second numerator and the second denominator divide out.

$$\frac{(x-7y)}{(x+5y)} \cdot \frac{(x-5y)\cancel{(x+2y)}}{(x-7y)\cancel{(x+2y)}}$$

$$\frac{(x-7y)}{(x+5y)} \cdot \frac{(x-5y)}{(x-7y)}$$

And the (x-7y) in the first numerator and second denominator divide out.

$$\frac{\cancel{(x-7y)}}{(x+5y)} \cdot \frac{(x-5y)}{\cancel{(x-7y)}}$$

What is left is the (x-5y) in the numerator and the (x+5y) in the denominator.

Final answer: $\dfrac{x-5y}{x+5y}$

P. 160 # 26.

$$\frac{x^2-8xy+16y^2}{x^2-3xy-10y^2} \cdot \frac{x^2-4y^2}{x^2-5xy+4y^2}$$

Solution: This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{(x-4y)(x-4y)}{(x-5y)(x+2y)} \cdot \frac{(x-2y)(x+2y)}{(x-4y)(x-y)}$$

Notice that one of the (x-4y) factors in the first numerator and the (x-4y) in the second denominator divide out.

$$\frac{(x-4y)\cancel{(x-4y)}}{(x-5y)(x+2y)} \cdot \frac{(x-2y)(x+2y)}{\cancel{(x-4y)}(x-y)}$$

$$\frac{(x-4y)}{(x-5y)(x+2y)} \cdot \frac{(x-2y)(x+2y)}{(x-y)}$$

Also the (x+2y) in the second numerator and the (x+2y) in first denominator divide out.

$$\frac{(x-4y)}{(x-5y)\cancel{(x+2y)}} \cdot \frac{(x-2y)\cancel{(x+2y)}}{(x-y)}$$

$$\frac{(x-4y)}{(x-5y)} \cdot \frac{(x-2y)}{(x-y)}$$

The final answer is $\frac{(x-4y)(x-2y)}{(x-5y)(x-y)}$.

If you want to, you can multiply out the numerator and denominator, but this is not necessary.

P. 160 # 28.

$$\frac{x^3y^2}{6xy+12x} \div \frac{y^3}{y^2-4}$$

Solution: This is a division problem, so you must invert the second fraction and multiply. Meanwhile, you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! In the first denominator, there are common factors of 6 and x to take out. The second denominator is a difference of two squares which factors and becomes the second numerator.

$$\frac{x^3y^2}{6x(y+2)} \cdot \frac{(y-2)(y+2)}{y^3}$$

Notice that the $(y+2)$ in the first denominator and the $(y+2)$ in the second numerator divide out.

$$\frac{x^3y^2}{6x\cancel{(y+2)}} \cdot \frac{(y-2)\cancel{(y+2)}}{y^3}$$

$$\frac{x^3y^2}{6x} \cdot \frac{(y-2)}{y^3}$$

Also the y^2 in the first numerator divides out with the y^3 in the second denominator, leaving a factor of y in the denominator.

$$\frac{x^3\cancel{y^2}}{6x} \cdot \frac{(y-2)}{\cancel{y^3}\,y}$$

$$\frac{x^3}{6x} \cdot \frac{(y-2)}{y}$$

Divide out the x factor, leaving

$$\frac{x^2}{6} \cdot \frac{(y-2)}{y}$$

The final answer is $\dfrac{x^2(y-2)}{6y}$

In the next few exercises, **FACTORING is the hardest part**. If you have trouble factoring these trinomials, then check out these links:

1. Advanced Factoring of Trinomials in Living C O L O R
2. One Step Explanation, Examples, and Exercises.

P. 161 # 30.

$$\frac{3x^2-x-2}{x^4-x^3} \div \frac{3x^2+5x+2}{x^3-2x^2}$$

Solution: This is a division problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out. In fact, the hardest part of this problem will be the factoring. See the links above.

$$\frac{(3x+2)(x-1)}{x^3(x-1)} \cdot \frac{x^2(x-2)}{(3x+2)(x+1)}$$

Notice that the $(x-1)$ in the first numerator and denominator divide out, and the $(3x+2)$ factor in the first numerator and the $(3x+2)$ in the second denominator divide out.

$$\frac{\cancel{(3x+2)}\ \cancel{(x-1)}}{x^3\ \cancel{(x-1)}} \cdot \frac{x^2(x-2)}{\cancel{(3x+2)}(x+1)}$$

$$\frac{1}{x^3} \cdot \frac{x^2(x-2)}{(x+1)}$$

Also, notice that the x^2 in the numerator divides out with the x^3 of the denominator, leaving an x in the denominator.

$$\frac{1}{x\,\cancel{x^3}} \cdot \frac{\cancel{x^2}(x-2)}{(x+1)}$$

This is the **final answer**: $\dfrac{x-2}{x(x+1)}$

In the next few exercises, **FACTORING is the hardest part**. If you have trouble factoring these trinomials, then check out these links:

3. Advanced Factoring of Trinomials in Living C O L O R
4. One Step Explanation, Examples, and Exercises.

P. 161 # 31. $\dfrac{9x^2-4y^2}{9x^2-12xy+4y^2} \div \dfrac{x^2-2xy-8y^2}{3x^2-14xy+8y^2}$

Solution: This is a division problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out. In fact, the hardest part of this problem will be the factoring.

$$\frac{(3x-2y)(3x+2y)}{(3x-2y)(3x-2y)} \bullet \frac{(3x-2y)(x-4y)}{(x-4y)(x+2y)}$$

Notice that the $(3x-2y)$ in the first numerator and denominator divide out. The $(3x-2y)$ factor in the first denominator and the $(3x-2y)$ in the second numerator divide out. Finally, the $(x-4y)$ in the second numerator and denominator divide out.

$$\frac{\cancel{(3x-2y)}(3x+2y)}{\cancel{(3x-2y)}\cancel{(3x-2y)}} \bullet \frac{\cancel{(3x-2y)}\cancel{(x-4y)}}{\cancel{(x-4y)}(x+2y)}$$

Final answer: $\dfrac{3x+2y}{x+2y}$

P. 161 # 32. $\dfrac{4x^2-9y^2}{4x^2-4xy-3y^2} \div \dfrac{4x^2+8xy+3y^2}{4x^2-y^2}$

Solution: This is a division problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out. In fact, the hardest part of this problem will be the factoring.

$$\frac{(2x-3y)(2x+3y)}{(2x+y)(2x-3y)} \bullet \frac{(2x-y)(2x+y)}{(2x+y)(2x+3y)}$$

Notice that the $(2x-3y)$ in the first numerator and denominator divide out. The $(2x+3y)$ factor in the first numerator and the $(2x+3y)$ in the second denominator divide out. Finally, the $(2x+y)$ in the second numerator and denominator divide out.

$$\frac{\cancel{(2x-3y)}\cancel{(2x+3y)}}{(2x+y)\cancel{(2x-3y)}} \bullet \frac{(2x-y)\cancel{(2x+y)}}{\cancel{(2x+y)}\cancel{(2x+3y)}}$$

Final Answer: $\dfrac{2x-y}{2x+y}$

P. 162 # 35.

$$\frac{25-x^2}{4x^3y} \div \frac{x^2-10x+25}{12xy^3}$$

Solution: This is a division problem, so you must invert the second fraction and multiply. Factor whenever possible in order to reduce the fractions in the next step.

$$\frac{(5-x)(5+x)}{4x^3y} \cdot \frac{12xy^3}{(x-5)(x-5)}$$

Notice that the (5-x) in the first numerator is the negative of the (x-5) in the second denominator. A number divided by its negative is -1, so these factors divide out, leaving a -1 in the first numerator.

$$\frac{-1\cancel{(5-x)}(5+x)}{4x^3y} \cdot \frac{12xy^3}{\cancel{(x-5)}(x-5)}$$

$$\frac{-1(5+x)}{4x^3y} \cdot \frac{12xy^3}{(x-5)}$$

Also, the 4 divides into the 12 leaving a factor of 3 in the numerator, and the x and the y divide out, leaving x^2 in the denominator and y^2 in the numerator

$$\frac{-1(5+x)}{x^2} \cdot \frac{3y^2}{(x-5)}$$

Final answer: $$\frac{-3y^2(x+5)}{x^2(x-5)}$$

P. 163 # 39. $\dfrac{x^3-27}{x^3+3x^2-9x-27} \bullet \dfrac{(x-3)^3}{x^2+3x+9}$

This is a multiplication problem, so of course you must factor whenever possible in order to reduce the fractions in the second step. However, the factoring might be tricky here. Notice that in the first fraction, the numerator is a difference of cubes, and the denominator requires factoring by grouping. In the second fraction, the numerator is already given in a factored form, and the denominator does NOT factor at all. In this problem, you must factor the first fraction, but leave the second fraction alone!

$$\frac{(x-3)(x^2+3x+9)}{x^2(x+3)-9(x+3)} \bullet \frac{(x-3)^3}{x^2+3x+9}$$

At this point, notice that the denominator is NOT factored yet, so continue by taking out the common factor of $(x+3)$.

$$\frac{(x-3)(x^2+3x+9)}{(x+3)(x^2-9)} \bullet \frac{(x-3)^3}{x^2+3x+9}$$

Now, notice that in the first denominator (x^2-9) is a difference of two squares, so this factors again.

$$\frac{(x-3)(x^2+3x+9)}{(x+3)(x-3)(x+3)} \bullet \frac{(x-3)^3}{x^2+3x+9}$$

Next, the factors indicated below in green divide out:

$$\frac{\cancel{(x-3)}\ \cancel{(x^2+3x+9)}}{(x+3)\ \cancel{(x-3)}\ (x+3)} \bullet \frac{(x-3)^3}{\cancel{x^2+3x+9}}$$

What is left when these factors are divided out is the final answer.

Final answer: $\dfrac{(x-3)^3}{(x+3)^2}$

P. 163 # 40. $\dfrac{x^2-25}{x^3-125} \div \dfrac{x^2-10x+25}{(x-5)^3}$

Solution: This is a division problem, so of course you must factor whenever possible in order to reduce the fractions, and invert the second fraction and multiply. However, the factoring might be tricky here. Notice that in the first fraction, the denominator is a difference of cubes. In the second fraction, the numerator is a perfect square trinomial, and the denominator is already given in a factored form.

$$\frac{(x-5)(x+5)}{(x-5)(x^2+5x+25)} \cdot \frac{(x-5)^3}{(x-5)(x-5)}$$

You might want to write out the second numerator as $(x-5)(x-5)(x-5)$

$$\frac{(x-5)(x+5)}{(x-5)(x^2+5x+25)} \cdot \frac{(x-5)(x-5)(x-5)}{(x-5)(x-5)}.$$

Next, the factors indicated below in green divide out:

$$\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x^2+5x+25)} \cdot \frac{(x-5)\cancel{(x-5)}\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x-5)}}$$

What is left when these factors are divided out is the final answer.

Final answer: $\dfrac{(x+5)(x-5)}{(x^2+5x+25)}$