Math in Living C O L O R !!

3.04 Rationalizing Denominators

Intermediate Algebra: One Step at a Time

Pages 259 - 262: #15, 16, 26.
Pages 264 - 266: #3, 5, 6, 8, 11, 12, 14, 20, 21, 22.

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See Section 3.04, with explanations, examples, and exercises, coming soon!
See Basic Algebra, with explanations, examples, and exercises, coming soon!

Definition: To **rationalize a denominator** means to eliminate the radicals from the denominator.

Rationalizing Monomial Denominators: Pages 259 – 262.

STEP 1: Simplify the radical.

STEP 2: Rationalize the denominator.

STEP 3: Reduce the fraction.

Rationalizing Binomial Denominators: Pages 264 – 266.

Multiply numerator and denominator by the "conjugate" of the denominator. That is, the same as the denominator, but with the opposite sign between the two terms.

P. 261. # 15. $\dfrac{40x^3}{\sqrt{20x^3}}$

SOLUTION.

STEP 1: Simplify the radical.

$$\frac{40x^3}{\sqrt{20x^3}}$$

$$\frac{40x^3}{\sqrt{4x^2}\sqrt{5x}}$$

$$\frac{40x^3}{2x\sqrt{5x}}$$

STEP 2: Rationalize the denominator.

Multiply the numerator and denominator by $\sqrt{5x}$

$$\frac{40x^3}{2x\sqrt{5x}}\frac{\bullet\sqrt{5x}}{\bullet\sqrt{5x}}$$

$$\frac{40x^3\sqrt{5x}}{2x\bullet 5x}$$

$$\frac{40x^3\sqrt{5x}}{10x^2}$$

STEP 3: Reduce the fraction.

Reduce the fraction by dividing out the $10x^2$.

$$\frac{4x\,\cancel{40x^3}\sqrt{5x}}{\cancel{10x^2}}$$

$$4x\sqrt{5x}$$

P. 261. # 16. $\dfrac{18x^5}{\sqrt{72x^7}}$

SOLUTION.

STEP 1: Simplify the radical.

$$\frac{18x^5}{\sqrt{72x^7}}$$

$$\frac{18x^5}{\sqrt{36x^6}\sqrt{2x}}$$

$$\frac{18x^5}{6x^3\sqrt{2x}}$$

STEP 2: Rationalize the denominator.

Multiply the numerator and denominator by $\sqrt{2x}$

$$\frac{18x^5}{6x^3\sqrt{2x}} \bullet \frac{\sqrt{2x}}{\sqrt{2x}}$$

$$\frac{18x^5 \bullet \sqrt{2x}}{6x^3 \bullet 2x}$$

$$\frac{18x^5 \bullet \sqrt{2x}}{12x^4}$$

STEP 3: Reduce the fraction.

Reduce the fraction by dividing out the $6x^4$.

$$\frac{3x\,\cancel{18}\,\cancel{x^5} \bullet \sqrt{2x}}{2\,\cancel{12}\,\cancel{x^4}}$$

$$\frac{3x\sqrt{2x}}{2}$$

P. 262. # 26. $\frac{35}{\sqrt[3]{7}}$

SOLUTION.

If the denominator is a cube root, you need to find a number that you can multiply times this radicand that will make it a perfect cube. Since you have a factor of 7 in the radical, remember, it will take three of a kind to make a perfect cube. This means that, since you already have one factor of 7, you need TWO more factors of 7 to make it a perfect cube. In other words, multiply numerator and denominator by $\sqrt[3]{7^2}$ or $\sqrt[3]{49}$.

$$\frac{35 \bullet \sqrt[3]{7^2}}{\sqrt[3]{7} \bullet \sqrt[3]{7^2}}$$

$$\frac{35 \bullet \sqrt[3]{7^2}}{\sqrt[3]{7^3}}$$

$$\frac{35 \bullet \sqrt[3]{7^2}}{7}$$

$$5 \bullet \sqrt[3]{49}$$

P. 263. # 3. $$\frac{15}{\sqrt{5}+5\sqrt{2}}$$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(\sqrt{5}-5\sqrt{2})$.

$$\frac{15}{\sqrt{5}+5\sqrt{2}}\,\frac{(\sqrt{5}-5\sqrt{2})}{(\sqrt{5}-5\sqrt{2})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{15(\sqrt{5}-5\sqrt{2})}{5-5\sqrt{10}+5\sqrt{10}-25\bullet 2}$$

The middle term subtracts out, and the 5 – 50 equals -45:

$$\frac{15(\sqrt{5}-5\sqrt{2})}{-45}$$

This reduces by dividing out the 15 with the -45:

$$\frac{(\sqrt{5}-5\sqrt{2})}{-3}$$

The tradition in math is to avoid negative denominators, so let's multiply numerator and denominator times -1.

$$\frac{-1\bullet(\sqrt{5}-5\sqrt{2})}{-1\bullet -3}.$$

Final answer: $\dfrac{-\sqrt{5}+5\sqrt{2}}{3}$ or $\dfrac{5\sqrt{2}-\sqrt{5}}{3}$

P. 264. # 5. $$\frac{12}{4+2\sqrt{3}}$$

Solution: Before beginning this problem, notice that you could factor the denominator and reduce the fraction. It turns out that this step is well worthwhile.

$$\frac{12}{2(2+\sqrt{3})}$$

Reduce the fraction by dividing out the 12 and the 2.

$$\frac{6}{(2+\sqrt{3})}$$

Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(2-\sqrt{3})$.

$$\frac{6}{(2+\sqrt{3})}\frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{6(2-\sqrt{3})}{4-2\sqrt{3}+2\sqrt{3}-3}$$

The middle term subtracts out, and the 4 – 3 equals 1

Final answer: $\frac{6(2-\sqrt{3})}{1}$ or $6(2-\sqrt{3})$ or $12-6\sqrt{3}$.

P. 264. # 6. $$\frac{12}{6-3\sqrt{3}}$$

Solution: Before beginning this problem, notice that you could factor the denominator and reduce the fraction. It turns out that this step is well worthwhile.

$$\frac{12}{3(2-\sqrt{3})}$$

Reduce the fraction by dividing out the 12 and the 3.

$$\frac{4}{(2-\sqrt{3})}$$

Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(2+\sqrt{3})$.

$$\frac{4}{(2-\sqrt{3})}\frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{4(2+\sqrt{3})}{4+2\sqrt{3}-2\sqrt{3}-3}$$

The middle term subtracts out, and the 4 – 3 equals 1

Final answer: $\frac{4(2+\sqrt{3})}{1}$ or $4(2+\sqrt{3})$ or $8+4\sqrt{3}$.

P. 264. # 8. $$\frac{6}{3\sqrt{2}+4\sqrt{3}}$$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(3\sqrt{2}-4\sqrt{3})$.

$$\frac{6}{(3\sqrt{2}+4\sqrt{3})}\frac{(3\sqrt{2}-4\sqrt{3})}{(3\sqrt{2}-4\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, but it's a good idea to leave the numerator in factored form.

$$\frac{6(3\sqrt{2}-4\sqrt{3})}{9\bullet 2-12\sqrt{6}+12\sqrt{6}-16\bullet 3}$$

The middle term always subtracts out!

$$\frac{6(3\sqrt{2}-4\sqrt{3})}{18-48}$$

Simplify the denominator

$$\frac{6(3\sqrt{2}-4\sqrt{3})}{-30}$$

Divide out the 6 and -30.

$$\frac{(3\sqrt{2}-4\sqrt{3})}{-5}$$

Avoid negative denominators, so multiply numerator and denominator by -1.

$$\frac{(-1)}{(-1)}\bullet\frac{(3\sqrt{2}-4\sqrt{3})}{-5}$$

Final answer: $\dfrac{-3\sqrt{2}+4\sqrt{3})}{5}$ or $\dfrac{4\sqrt{3}-3\sqrt{2}}{5}$

P. 265. #11. $$\frac{\sqrt{20}}{6-\sqrt{6}}$$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(6+\sqrt{6})$.

$$\frac{\sqrt{20}}{6-\sqrt{6}}\frac{(6+\sqrt{6})}{(6+\sqrt{6})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify $\sqrt{20}=\sqrt{4}\sqrt{5}$ or $2\sqrt{5}$

$$\frac{2\sqrt{5}(6+\sqrt{6})}{36+6\sqrt{6}-6\sqrt{6}-6}$$

The middle term always subtracts out!

$$\frac{12\sqrt{5}+2\bullet\sqrt{30}}{36-6}$$

Simplify the numerator and denominator.

$$\frac{12\sqrt{5}+2\bullet\sqrt{30}}{30}$$

$$\frac{2(6\sqrt{5}+\sqrt{30})}{30}$$

$$\frac{\cancel{2}(6\sqrt{5}+\sqrt{30})}{\cancel{30}\ 15}$$

Final answer: $$\frac{6\sqrt{5}+\sqrt{30}}{15}$$

Since this is a numerical problem, you can check with your calculator by calculating the decimal approximation of the problem and of the answer to see if they agree.

$$\frac{\sqrt{20}}{6-\sqrt{6}} = 1.259575563$$

$$\frac{6\sqrt{5}+\sqrt{30}}{15} = 1.259575563$$ It checks!!

P. 265. #12. $$\frac{\sqrt{20}}{6+\sqrt{6}}$$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(6-\sqrt{6})$.

$$\frac{\sqrt{20}}{6+\sqrt{6}}\,\frac{(6-\sqrt{6})}{(6-\sqrt{6})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify $\sqrt{20}=\sqrt{4}\sqrt{5}$ or $2\sqrt{5}$.

$$\frac{2\sqrt{5}(6-\sqrt{6})}{36-6\sqrt{6}+6\sqrt{6}-6}$$

The middle term always subtracts out!

$$\frac{12\sqrt{5}-2\bullet\sqrt{30}}{36-6}$$

Simplify the numerator and denominator.

$$\frac{12\sqrt{5}-2\bullet\sqrt{30}}{30}$$

$$\frac{2(6\sqrt{5}-\sqrt{30})}{30}$$

$$\frac{\cancel{2}(6\sqrt{5}-\sqrt{30})}{\cancel{30}\ 15}$$

$$\frac{6\sqrt{5}-\sqrt{30}}{15}$$

Since this is a numerical problem, you can check with your calculator by calculating the decimal approximation of the problem and of the answer to see if they agree.

$$\frac{\sqrt{20}}{6+\sqrt{6}} = 0.5292788193$$

$$\frac{6\sqrt{5}-\sqrt{30}}{15} = 0.5292788193$$ It checks!!

P. 265. #14. $$\frac{\sqrt{27}}{2\sqrt{6}-3\sqrt{3}}$$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(2\sqrt{6}+3\sqrt{3})$.

$$\frac{\sqrt{27}}{(2\sqrt{6}-3\sqrt{3})}\,\frac{(2\sqrt{6}+3\sqrt{3})}{(2\sqrt{6}+3\sqrt{3})}$$

It is best to multiply out (FOIL) the denominator, and since there are also radicals in the numerator, it will be necessary to multiply out the numerator as well. It might be helpful to go ahead and simplify $\sqrt{27}=\sqrt{9}\sqrt{3}$ or $3\sqrt{3}$.

$$\frac{3\sqrt{3}(2\sqrt{6}+3\sqrt{3})}{4\bullet 6+6\sqrt{18}-6\sqrt{18}-9\bullet 3}$$

The middle term always subtracts out!

$$\frac{6\sqrt{18}+9\bullet 3}{24-27}$$

Simplify the numerator and denominator.

$$\frac{6\sqrt{9}\sqrt{2}+27}{-3}$$

$$\frac{6\bullet 3\sqrt{2}+27}{-3}$$

$$\frac{18\sqrt{2}+27}{-3}$$

$$\frac{9(2\sqrt{2}+3)}{-3}$$

Final answer: $-3(2\sqrt{2}+3)$ or $-6\sqrt{2}-9$

P. 266. #20. $\dfrac{\sqrt{6}-\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(\sqrt{6}-\sqrt{3})$.

$$\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}+\sqrt{3}} \bullet \frac{(\sqrt{6}-\sqrt{3})}{(\sqrt{6}-\sqrt{3})}$$

In this case, you must multiply out (F OI L) both the numerator and the denominator.

$$\frac{6-\sqrt{18}-\sqrt{18}+3}{6-\sqrt{18}+\sqrt{18}-3}$$

Simplify numerator and denominator.

$$\frac{9-2\sqrt{18}}{3}$$

$$\frac{9-2\sqrt{9}\bullet\sqrt{2}}{3}$$

$$\frac{9-6\bullet\sqrt{2}}{3}$$

Factor the numerator in order to reduce the fraction.

$$\frac{\cancel{3}(3-2\bullet\sqrt{2})}{\cancel{3}}$$

Final answer: $3-2\bullet\sqrt{2}$

This is a great place to check with the calculator! Use your calculator to calculate the problem. Be sure to put parentheses around the numerator and denominator, and if your calculator opens parentheses when you enter square root, be sure to close the parentheses after you enter the number!

Problem = $\dfrac{\left(\sqrt{6}-\sqrt{3}\right)}{\left(\sqrt{6}+\sqrt{3}\right)}$ should give you .1715728753 . . .

Answer = $3-2\bullet\sqrt{2}$ also should give you .1715728753 . . .

This checks!!

P. 266. #21. $\dfrac{4\sqrt{5}+5\sqrt{2}}{3\sqrt{2}-2\sqrt{5}}$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(3\sqrt{2}+2\sqrt{5})$.

$$\frac{4\sqrt{5}+5\sqrt{2}}{3\sqrt{2}-2\sqrt{5}}\,\frac{(3\sqrt{2}+2\sqrt{5})}{(3\sqrt{2}+2\sqrt{5})}$$

In this case, you must multiply out (F OI L) both the numerator and the denominator.

$$\frac{12\bullet\sqrt{10}+8\sqrt{25}+15\sqrt{4}+10\bullet\sqrt{10}}{9\bullet 2+6\sqrt{10}-6\sqrt{10}-4\bullet 5}$$

Simplify numerator and denominator.

$$\frac{22\sqrt{10}+8\bullet 5+15\bullet 2}{18-20}$$

$$\frac{22\sqrt{10}+40+30}{-2}$$

$$\frac{22\sqrt{10}+70}{-2}$$

Factor the numerator in order to reduce the fraction.

$$\frac{2(11\sqrt{10}+35)}{-2}$$

Final answer: $-1(11\sqrt{10}+35)$ or $-11\sqrt{10}-35$

Finally, as a check, if you calculate the decimal approximation of the problem and of the answer, you should get approximately -69.78505.

P. 266. #22. $\dfrac{3\sqrt{10}-2\sqrt{6}}{4\sqrt{10}+5\sqrt{6}}$

Solution: Rationalize the denominator by multiplying numerator and denominator of the fraction by the conjugate of the denominator. That is the same as the denominator by with opposite sign: $(4\sqrt{10}-5\sqrt{6})$.

$$\frac{(3\sqrt{10}-2\sqrt{6})}{(4\sqrt{10}+5\sqrt{6})}\,\frac{(4\sqrt{10}-5\sqrt{6})}{(4\sqrt{10}-5\sqrt{6})}$$

In this case, you must multiply out (F OI L) both the numerator and the denominator.

$$\frac{12\bullet 10-15\sqrt{60}-8\sqrt{60}+10\bullet 6}{16\bullet 10-20\sqrt{60}+20\sqrt{60}-25\bullet 6}$$

Simplify numerator and denominator.

$$\frac{120-23\sqrt{60}+60}{160-150}$$

$$\frac{180-23\sqrt{4}\sqrt{15}}{10}$$

$$\frac{180-23\bullet 2\sqrt{15}}{10}$$

$$\frac{180-46\sqrt{15}}{10}$$

Factor the numerator in order to reduce the fraction.

$$\frac{\cancel{2}(90-23\sqrt{15})}{\cancel{10}\ 5}$$

Final answer: $\dfrac{90-23\sqrt{15}}{5}$