Math in Living C O L O R !!

3.02 Operations with Radicals

Intermediate Algebra: One Step at a Time.
Page 250- 255: #16, 17, 19, 20, 21, 23, 24, 38, 41, 42, 44, 45, 48, 51, 52, 55, 58, 60, 64

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See Section 3.02, with detailed explanations, examples, and exercises, coming soon!

Radicals are not as hard as you think they are! However, before you do anything with a cube root, you must have these special numbers in mind (or on a piece of paper!) in front of you:
$2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$.

Memorize them: 8, 27, 64, 125

Before you do anything with a 4^{th} root, be thinking $2^4=16$ or $3^4=81$.
A 5^{th} root problem will almost always involve $2^5=32$.

P. 250: #16. $5\sqrt[3]{108} - 4\sqrt[3]{32}$

Solution: Find a perfect cube that divides into 108 (that would be 27) and a perfect cube that divides into 32 (that would be 8). $108=27\bullet 4$ and $32=8\bullet 4$.

$5\sqrt[3]{108} - 4\sqrt[3]{32}$
$5\sqrt[3]{27\bullet 4} - 4\sqrt[3]{8\bullet 4}$
$5\sqrt[3]{27}\bullet\sqrt[3]{4} - 4\sqrt[3]{8}\bullet\sqrt[3]{4}$
$5\bullet 3\bullet\sqrt[3]{4} - 4\bullet 2\bullet\sqrt[3]{4}$

Now multiply the numbers 5 times 3 and the 4 times 2.
$15\sqrt[3]{4} - 8\sqrt[3]{4}$

Now you have like terms so you can combine the 15 and the -8, and the final answer is $7\sqrt[3]{4}$

In # 17, you have 4th roots, so keep in mind that $2^4=16$ and $3^4=81$.

P. 250: #17. $7\sqrt[4]{32} - 3\sqrt[4]{162}$

Solution: Find a perfect 4th power that divides into 32 (that would be 16) and a perfect 4th power that divides into 162 (that would be 81). $32=16\cdot 2$ and $162=81\cdot 2$.

$$7\sqrt[4]{32} - 3\sqrt[4]{162}$$

$$7\sqrt[4]{16\bullet 2} - 3\sqrt[4]{81\bullet 2}$$

$$7\sqrt[4]{16}\bullet\sqrt[4]{2} - 3\sqrt[4]{81}\bullet\sqrt[4]{2}$$

$$7\bullet 2\bullet\sqrt[4]{2} - 3\bullet 3\bullet\sqrt[4]{2}$$

Now multiply the numbers 7 times 2 and the 3 times 3.

$$14\sqrt[4]{2} - 9\sqrt[4]{2}$$

Now you have like terms, so combine the 14 and the −9. The final answer is $5\sqrt[4]{2}$

P. 251: #19. $7x^2\sqrt{24xy^6} + 8y^3\sqrt{54x^5}$

Solution: $7x^2\sqrt{}\sqrt{} + 8y^3\sqrt{}\sqrt{}$

First, separate each of the square roots into two square roots. Sort out the square roots into perfect squares that go in the first (red) square root, and the left-over factors that go in the second (blue) square root.

$$7x^2\sqrt{4y^6}\sqrt{6x} + 8y^3\sqrt{9x^4}\sqrt{6x}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$7x^2\bullet 2y^3\sqrt{6x} + 8y^3\bullet 3x^2\sqrt{6x}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$14x^2y^3\sqrt{6x} + 24x^2y^3\sqrt{6x}$$

Notice that these are like radicals and like terms. They combine together:

$$38x^2y^3\sqrt{6x}$$

P. 251: #20. $5xy\sqrt{20x^7y^5} - 4\sqrt{45x^9y^7}$

Solution: $5xy\sqrt{\quad}\sqrt{\quad} - 4\sqrt{\quad}\sqrt{\quad}$

First, separate each of the square roots into two square roots. Sort out the square roots into perfect squares that go in the first (red) square root, and the left-over factors that go in the second (blue) square root.

$$5xy\sqrt{4x^6y^4}\sqrt{5xy} - 4\sqrt{9x^8y^6}\sqrt{5xy}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$5xy \bullet 2x^3y^2\sqrt{5xy} - 4 \bullet 3x^4y^3\sqrt{5xy}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$10x^4y^3\sqrt{5xy} - 12x^4y^3\sqrt{5xy}$$

Notice that these are like radicals and like terms. They combine together:

$$-2x^4y^3\sqrt{5xy}$$

P. 251: #21. $3x^2y\sqrt{20xy^4} - 2x\sqrt{45x^3y^6}$

Solution: $3x^2y\sqrt{\quad}\sqrt{\quad} - 2x\sqrt{\quad}\sqrt{\quad}$

First, separate each of the square roots into two square roots. Sort out the square roots into perfect squares that go in the first (red) square root, and the left-over factors that go in the second (blue) square root.

$$3x^2y\sqrt{4y^4}\sqrt{5x} - 2x\sqrt{9x^2y^6}\sqrt{5x}$$

Everyone can take the square root of the first (red) radicals since they are perfect squares. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$3x^2y \bullet 2y^2\sqrt{5x} - 2x \bullet 3xy^3\sqrt{5x}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$6x^2y^3\sqrt{5x} - 6x^2y^3\sqrt{5x}$$

Notice that these are like radicals and like terms. In fact, they subtract out. The final answer is

$$0$$

P. 251: #23. $5x^2y\sqrt[3]{54x^7y^5} - 4xy^2\sqrt[3]{16x^{10}y^2}$

Solution: $5x^2y\sqrt[3]{\underline{\qquad}}\sqrt[3]{\underline{\qquad}} - 4xy^2\sqrt[3]{\underline{\qquad}}\sqrt[3]{\underline{\qquad}}$

First, separate each of the cube roots into two cube roots. Sort out each cube roots into perfect cubes that go in the first (red) cube root, and the left-over factors that go in the second (blue) cube root.

$$5x^2y\sqrt[3]{27x^6y^3}\sqrt[3]{2xy^2} - 4xy^2\sqrt[3]{8x^9}\sqrt[3]{2xy^2}$$

Everyone can take the cube root of the first (red) radicals since they are perfect cubes. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$5x^2y \bullet 3x^2y\sqrt[3]{2xy^2} - 4xy^2 \bullet 2x^3\sqrt[3]{2xy^2}$$

Do what you are able to do next--multiply outside the radicals:

$$15x^4y^2\sqrt[3]{2xy^2} - 8x^4y^2\sqrt[3]{2xy^2}$$

Notice that these are like radicals and like terms. Combining like terms, you have

$$7x^4y^2\sqrt[3]{2xy^2}$$ **Final Answer!!**

P. 251: #24. $7x\sqrt[3]{16xy^3} + 8y\sqrt[3]{54x^4}$

Solution: $7x\sqrt[3]{\underline{\qquad}}\sqrt[3]{\underline{\qquad}} + 8y\sqrt[3]{\underline{\qquad}}\sqrt[3]{\underline{\qquad}}$

First, separate each of the cube roots into two cube roots. Sort out each cube roots into perfect cubes that go in the first (red) cube root, and the left-over factors that go in the second (blue) cube root.

$$7x\sqrt[3]{8y^3}\sqrt[3]{2x} + 8y\sqrt[3]{27x^3}\sqrt[3]{2x}$$

Everyone can take the cube root of the first (red) radicals since they are perfect cubes. Nobody knows what to do about the second (blue) radical since they cannot be simplified. So do what you can do (the red radicals), and leave the rest (blue radicals!) alone:

$$7x \bullet 2y\sqrt[3]{2x} + 8y \bullet 3x\sqrt[3]{2x}$$

As in the last step, you do what you are able to do next--multiply outside the radicals:

$$14xy\sqrt[3]{2x} + 24xy\sqrt[3]{2x}$$

Notice that these are like radicals and like terms. Combining like terms, you have

$$38xy\sqrt[3]{2x}$$ **Final Answer!!**

P. 252: #38. $\sqrt{155} \bullet \sqrt{124}$

Solution: First, you can multiply the numbers together and place the result within a single square root. However, if you do this, you will get one very large number that will be difficult to break down. So, instead of multiplying $155 \bullet 124$, write the numbers in factored form:

$$\sqrt{5 \bullet 31 \bullet 4 \bullet 31}$$

Next, notice that something "special" happened in this last step! You have a "pair" of 31 factors. Now, you can rewrite this as a product of two radicals in which you place the perfect squares in the first (red) radical, and the "left over" factor(s) in the second (blue) radical. The trick here is to recognize that the pair of 31 factors is a square, but also the 4 is a perfect square, and these must all go in the first (red) radical.

$$\sqrt{4 \bullet 31^2} \bullet \sqrt{5}$$

$$2 \bullet 31 \sqrt{5}$$

$$62\sqrt{5}$$ **Final Answer!!**

Since this is a numerical problem, it can be checked with a calculator.

Calculate the value of the problem: $\sqrt{155} \bullet \sqrt{124}$ = 138.6363146

Calculate the value of the answer: $62\sqrt{5}$ = 138.6363146

P. 253: #41. $\sqrt[3]{75} \bullet \sqrt[3]{15}$

Solution: First, you can multiply the numbers together and place the result within a single cube root. However, if you do this, you will get one very large number that will be difficult to break down. So, instead of multiplying $75 \bullet 15$, write the numbers in factored form:

$$\sqrt[3]{3 \bullet 25 \bullet 3 \bullet 5}$$

$$\sqrt[3]{3 \bullet 5 \bullet 5 \bullet 3 \bullet 5}$$

Next, sort out these factors, placing any perfect cubes (that is, three of a kind!) first (red) cube root, and place the left-over factors in the second (blue) cube root. Notice that you have three factors of 5, so put these in the first cube root. What is "left-over" is a pair of 3's, so they go in the second cube root.

$$\sqrt[3]{5 \bullet 5 \bullet 5} \bullet \sqrt[3]{3 \bullet 3}$$

$$\sqrt[3]{5^3} \bullet \sqrt[3]{9}$$

$$5 \bullet \sqrt[3]{9}$$ **Final Answer!!**

Since this is a numerical problem, it can be checked with a calculator.

Calculate the value of the problem: $\sqrt[3]{75} \bullet \sqrt[3]{15}$ = 10.40041912

Calculate the value of the answer: $5 \bullet \sqrt[3]{9}$ = 10.40041912.

P. 253: #42. $\sqrt[3]{105} \cdot \sqrt[3]{45}$

Solution: First, you can multiply the numbers together and place the result within a single cube root. However, if you do this, you will get one very large number that will be difficult to break down. So, instead of multiplying $105 \cdot 45$, write the numbers in factored form:

$$\sqrt[3]{3 \cdot 35 \cdot 9 \cdot 5}$$

$$\sqrt[3]{3 \cdot 5 \cdot 7 \cdot 3 \cdot 3 \cdot 5}$$

Next, sort out these factors, placing any perfect cubes (that is, three of a kind!) first (red) cube root, and place the left-over factors in the second (blue) cube root. Notice that you have three factors of 3, so put these in the first cube root.

$$\sqrt[3]{3 \cdot 3 \cdot 3} \cdot \sqrt[3]{5 \cdot 7 \cdot 5}$$

$$\sqrt[3]{3^3} \cdot \sqrt[3]{175}$$

$$3 \cdot \sqrt[3]{175}$$ Final Answer!!

Since this is a numerical problem, it can be checked with a calculator.

Calculate the value of the problem: $\sqrt[3]{105} \cdot \sqrt[3]{45}$ = 16.78033413

Calculate the value of the answer: $3 \cdot \sqrt[3]{175}$ = 16.78033413

P. 253: #45. $4\sqrt{3} \cdot 6\sqrt{15}$

Solution: Remember, you multiply the numbers that are OUTSIDE the radical together, and you keep them OUTSIDE the radical. Then you multiply the numbers that are INSIDE the radical together and keep them INSIDE the radical.

$$24\sqrt{45}$$

Now, simplify the radical 45. Break it down into 9 times 5.

$$24\sqrt{9}\sqrt{5}$$

$$24 \cdot 3\sqrt{5}$$

$$72\sqrt{5}$$ Final Answer!!

Since this is a numerical problem, you can check the answer by calculating the value of the problem:

$4\sqrt{3} \cdot 6\sqrt{15}$ = 160.9968944 . . .

and the value of the answer that you obtained:

$72\sqrt{5}$ = 160.9968944 . . .

P. 253: #48. $8\sqrt[3]{65} \bullet 2\sqrt[3]{50}$

Solution: Remember, you multiply the numbers that are **OUTSIDE** the radical together, and you keep them **OUTSIDE** the radical. Then you multiply the numbers that are **INSIDE** the radical together and keep them **INSIDE** the radical.

$$8 \bullet 2\sqrt[3]{65 \bullet 50}$$

However, if you use a calculator and multiply out the numbers that are INSIDE the radical, you end up with a large number that you won't know how to simplify. It's better, instead of multiplying the numbers out, to break them down into prime factors, and for square roots, look for pairs of numbers, for cube roots, look for three of a kind, etc.

$$8 \bullet 2\sqrt[3]{5 \bullet 13 \bullet 5 \bullet 10}$$

$$16\sqrt[3]{5 \bullet 13 \bullet 5 \bullet 5 \bullet 2}$$

Notice that you have three factors of 5! That makes a perfect cube:

$$16\sqrt[3]{5^3 \bullet 13 \bullet 2}$$

Now, separate into two radicals, with the perfect cube in the first, and the leftover factors in the second radical.

$$16 \bullet 5 \bullet \sqrt[3]{13 \bullet 2}$$

$80\sqrt[3]{26}$ **Final Answer!!**

Since this is a numerical problem, you may want to check the answer by calculating the problem:

$$8\sqrt[3]{65} \bullet 2\sqrt[3]{50} = 236.9996855\ldots$$

and then calculate the answer that you obtained:

$$80\sqrt[3]{26} = 236.9996855\ldots$$

(Note: as I was working this problem for you, I made a mistake and I missed it! Because of this check, I had to go back and find an error of my own!!)

P. 254: #51. $8\sqrt{10}\,(2\sqrt{6}-3\sqrt{2})$

Solution: This is a problem that uses the Distributive Property. You must multiply $8\sqrt{10}\bullet 2\sqrt{6}$ and $8\sqrt{10}\bullet 3\sqrt{2}$. As always, you multiply the numbers that are OUTSIDE the radical together, and you keep them OUTSIDE the radical. Then you multiply the numbers that are INSIDE the radical together and keep them INSIDE the radical.

$$16\sqrt{60}-24\sqrt{20}$$

Now, simplify $\sqrt{60}$ (break it down into 4 times 15), and $\sqrt{20}$ (break it down into 4 times 5).

$$16\sqrt{60}-24\sqrt{20}$$

$$16\sqrt{4}\sqrt{15}-24\sqrt{4}\sqrt{5}$$

$$16\bullet 2\sqrt{15}-24\bullet 2\sqrt{5}$$

$$32\sqrt{15}-48\sqrt{5}$$ **Final Answer!!**

You might want to check the answer by calculating the value of the problem:

$8\sqrt{10}\,(2\sqrt{6}-3\sqrt{2})$ = 16.60420416 . . .

and compare to the value of the answer that you obtained:

$32\sqrt{15}-48\sqrt{5}$ = 16.60420416 . . .

Note: With the calculator, be careful to close parentheses whenever the calculator opens them!

P. 254: #52. $2\sqrt{6}\,(4\sqrt{3}+5\sqrt{2})$

Solution: This is a problem that uses the Distributive Property. You must multiply the $2\sqrt{6}\bullet 4\sqrt{3}$ and $2\sqrt{6}\bullet 5\sqrt{2}$. As always, you multiply the numbers that are OUTSIDE the radical together, and you keep them OUTSIDE the radical. Then you multiply the numbers that are INSIDE the radical together and keep them INSIDE the radical.

$$8\sqrt{18}+10\sqrt{12}$$

Now, simplify $\sqrt{18}$ (break it down into 9 times 2), and $\sqrt{12}$ (break it down into 4 times 3).

$$8\sqrt{18}+10\sqrt{12}$$

$$8\sqrt{9}\sqrt{2}+10\sqrt{4}\sqrt{3}$$

$$8\bullet 3\sqrt{2}+10\bullet 2\sqrt{3}$$

$$24\sqrt{2}+20\sqrt{3}$$ **Final Answer!!**

You might want to check the answer by calculating the value of the problem:

$2\sqrt{6}\,(4\sqrt{3}+5\sqrt{2})$ = 68.58214165 . . .

and compare to the value of the answer that you obtained:

$24\sqrt{2}+20\sqrt{3}$ = 68.58214165. . .

Note: With the calculator, be careful to close parentheses whenever the calculator opens them!

P. 254: #55.

$$\left(4+5\sqrt{6}\right)\left(8+2\sqrt{6}\right)$$

Solution:

F O I L

$$4\bullet 8+8\sqrt{6}+40\sqrt{6}+10\bullet 6$$

$$32 + 48\sqrt{6} + 60$$

$$92 + 48\sqrt{6}$$

As a check, calculate the value of the problem: $\left(4+5\sqrt{6}\right)\left(8+2\sqrt{6}\right)$ = 209.5755077

Then, calculate the value of the answer: $92 + 48\sqrt{6}$ = 209.5755077

Note: With the calculator, be careful to close parentheses whenever the calculator opens them!

P. 254: #58.

$$\left(4\sqrt{5}-5\sqrt{15}\right)\left(3\sqrt{5}+2\sqrt{15}\right)$$

Solution:

F O I L

$$12\bullet 5+8\sqrt{75}-15\sqrt{75}-10\bullet 15$$

$$60 - 7\sqrt{75} - 150$$

$$-90 - 7\sqrt{25}\sqrt{3}$$

$$-90 - 7\bullet 5\sqrt{3}$$

$$-90 - 35\sqrt{3}$$

As a check, calculate the value of the problem: $\left(4\sqrt{5}-5\sqrt{15}\right)\left(3\sqrt{5}+2\sqrt{15}\right)$ = -150.6217783

Then, calculate the value of the answer: $-90 - 35\sqrt{3}$ = -150.6217783

Note: With the calculator, be careful to close parentheses whenever the calculator opens them!

P. 255: #60.

$$\left(4\sqrt{6}+5\sqrt{2}\right)^2$$

Solution:

$$\left(4\sqrt{6}+5\sqrt{2}\right)\left(4\sqrt{6}+5\sqrt{2}\right)$$

F O I L

$$16\bullet 6+20\sqrt{12}+20\sqrt{12}+25\bullet 2$$

$$96 + 40\sqrt{12} + 50$$

$$146 + 40\sqrt{4}\sqrt{3}$$

$$146 + 40\bullet 2\sqrt{3}$$

$$146 + 80\sqrt{3}$$

As a check, calculate the value of the problem: $\left(4\sqrt{6}+5\sqrt{2}\right)^2$ = 284.5640646 . . .

Then, calculate the value of the answer: $146 + 80\sqrt{3}$ = 284.5640646 . . .

Note: With the calculator, be careful to close parentheses whenever the calculator opens them!

P. 255: #64. $\left(5+\sqrt[3]{5}\right)\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$

Solution: Multiply the first (5) times everything in the second parentheses:

$$5\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$125 - 25\sqrt[3]{5} + 5\sqrt[3]{25}$$

Next, multiply the second ($\sqrt[3]{5}$) times everything in the second parentheses:

$$\sqrt[3]{5}\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$25\sqrt[3]{5}-5\sqrt[3]{25}+\sqrt[3]{125}$$

Now, put it ALL together and combine like terms:

$$\left(5+\sqrt[3]{5}\right)\left(25-5\sqrt[3]{5}+\sqrt[3]{25}\right)$$

$$= 125 - 25\sqrt[3]{5} + 5\sqrt[3]{25}$$

$$+ 25\sqrt[3]{5}-5\sqrt[3]{25}+\sqrt[3]{125}$$

$$= 125 + \sqrt[3]{125}$$

$$= 125 + 5$$

$$= 130$$

Joke for the Day:

What does this fraction mean $\frac{\text{NaCl NaCl}}{\text{CCCCCCC}}$ **???**

For answer, scroll down!

Saline, Saline, over the 7 "C's" !!