Math in Living C O L O R !!

5.06 Functional Notation, Functions, Domain, and Range

Intermediate Algebra: One Step at a Time, Pages 427 - 446:

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See Section 5.06, with explanations, examples, and exercises, coming soon!

Functions, Domain, Range Summary

Domain--Set of all (permissible) x values

Range--Set of all (resulting) y values

**Relation--any graph (in which two variables are "related").**

Function--a relation in which each value of x has a unique value of y.

To find the Domain, solve for y in terms of x (or look at the graph!).

To find the Range, solve for x in terms of y (or look at the graph!).

Domain and Range (from a Graph)

Example 1: Find the domain and range for the graph sketched below. You may recognize that this is a parabola that opens downward with vertex at **(0, −9)**.

Y1=-X2-9

X=0   Y=-9
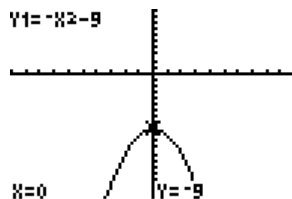
Window: **x=[-10,10] y=[-20,10]**

Solution: The sketch above has a vertex at **y=-9**. From this point, the graph extends all the way to the left and all the way to the right. It also extends from **y=-9**, all the way down on the graph. The graph never goes above **y=-9**. Therefore, the domain is all real values. The range is all values BELOW the vertex--that is, $y \leq -9$ or $(-\infty, -9]$.

**Domain: All Real Numbers or (−∞,∞).**   **Range:** $(-\infty, -9]$

Example 2: Find the domain and range for the graph sketched below.

Y1=√(X+6)

X=-6   Y=0
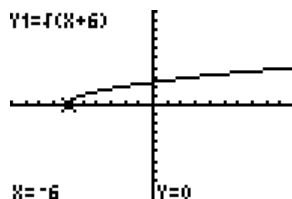
Solution: In this graph you may recognize the upper half of a parabola that extends upward and to the right to infinity. The x-values of this graph extend from the vertex at **x=-6**, all the way to the right side of the graph and beyond. Therefore, the domain is $[-6, \infty)$. The range is all values that are on or above the x-axis, or $y \geq 0$, or in interval notation $[0, \infty)$.

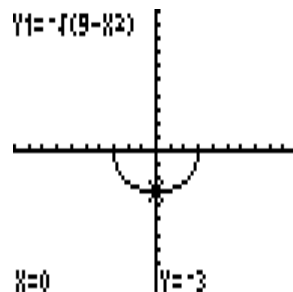**Domain:** $[-6, \infty)$ **Range:** $[0, \infty)$

Example 3: Find the domain and range for the graph sketched below.

Y1=-√(X+5)

X=-5   Y=0
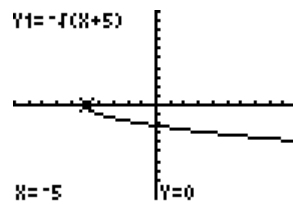
Solution: You may recognize that this is the lower half of a parabola that extends downward and to the right to infinity. The domain is $[-5,\infty)$. The range is all values below the x-axis, which can be written $y \leq 0$, or in interval notation $(-\infty,0]$.

Domain: $[-5,\infty)$ Range: $(-\infty,0]$

Example 4: Find the domain and range for the graph sketched below.

Y1=-√(9-X2)

X=0   Y=-3

Solution: From this graph, you may recognize this as the lower half of a circle that extends 3 units down, 3 units to the right, and 3 units to the left. The domain is $[-3,3]$. The range is all values from −3 up to 0, or in interval notation $[-3,0]$.

Domain: [−3,3]   Range: [−3,0]

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Example 5: Find the domain and range for the graph sketched below.

Y1=√(X2-5X-6)   Y1=√(X2-5X-6)

X=-1   Y=0   X=6   Y=0
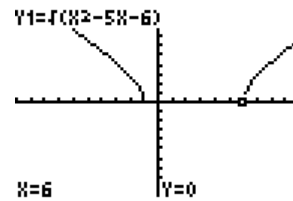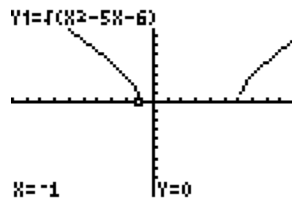
Solution: In the two sketches above, it is clear that there are points at (-1,0) and (6,0) that will be critical to finding the domain and range.   Notice that the graph actually touches the x axis at these two points, and from these points the graph extends upward from y=0. The graph extends to the left from x=-1, and to the right from x=6.

From these graphs, it should be clear that the domain is $(-\infty, -1] \cup [6, \infty)$. Likewise, the range is all values that are on or above the x-axis, or $y \geq 0$, or in interval notation $[0, \infty)$.

Domain: $(-\infty, -1] \cup [6, \infty)$ Range: $[0, \infty)$

Example 6: Find the domain and range for the graph sketched below.
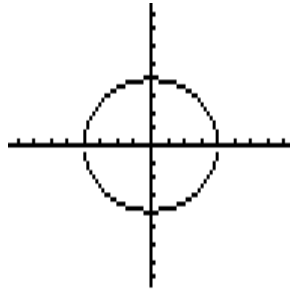
Y1=-√(X2-16)

X=4   Y=0
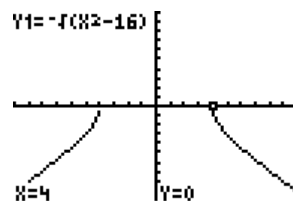
Solution: Can you see that the graph extends (left to right) in the x-direction from negative infinity to -4 and from 4 to infinity? Therefore the domain is $(-\infty, -4] \cup [4, \infty)$. The values of y extend from negative infinity up to zero, so the range is $(-\infty, 0]$.

Domain: $(-\infty, -4] \cup [4, \infty)$ Range: $(-\infty, 0]$

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**Example 7:** Find the domain and range for the graph sketched below.

Solution: You can see that this is the graph of a circle whose center is at the origin and whose radius is 4. You can also see that the values of x extend from -4 to 4 inclusive, and the values of y also extend from -4 to 4 inclusive.

Domain: $[-4,4]$

Range: $[-4,4]$

Example 8: Find the domain and range for the graph that is sketched below.
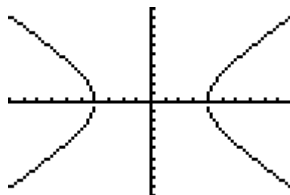
Solution: The domain consists of all values to left of and including -4 and to the right of and including 4 $(-\infty,-4]\cup[4,\infty)$. The range extends all the way down to negative infinity and all the way up to positive infinity. This would be all real values or $(-\infty,\infty)$.

Domain: $(-\infty,-4]\cup[4,\infty)$

Range: $(-\infty,\infty)$