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MAC 1143 PRECALCULUS FINAL EXAM A NAME _____

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. *SHOW ALL STEPS!!*
Explain your calculations and procedures for partial credit.
TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.
Describe windows, give key strokes, show what you did!!
*IN EACH EXERCISE, ALL SET UP ALL EQUATIONS BEFORE SOLVING!!!*

1. Sketch and label two periods of the function $Y = 40 \sin 50t$. Give the amplitude, period, and phase shift.

2. Solve the equations for t in the interval [0, 360]:

   a) $\cos t = -0.5$ b) $t = \arccos(-0.5)$

   Are these the same or different? Explain.

3. Express the following in terms of one function of one angle (or constant!):

   a) $\sin 36° \cos 4° + \cos 36° \sin 4° =$ _____

   b) $\cos 36° \cos 4° + \sin 36° \sin 4° =$ _____

   c) $1 - \cos^2 48° =$ _____

   d) $1 - 2\cos^2 48° =$ _____

   e) $\cos^2 48° + \sin^2 48° =$ _____

In 4 - 5, prove the trigonometric identities by classroom methods:

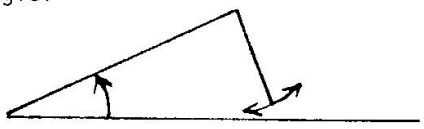
4. $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} = 2 \tan x \sec x$

5. $\sin 3x = 3 \sin x - 4 \sin^3 x$

6. Use the method of your choice to find all solutions in the interval $[0, 2\pi)$. Justify your answer. Explain your method and why you selected it.

$$\sec^2 x + 2 \sec x - 8 = 0$$

7. Given a SSA triangle with $B = 35°$ and $c = 69$. For what value of "b" would there be exactly one <u>right</u> triangle. Label the figure below with appropriate angles and sides showing the "fixed" and "hinged" sides. Under what conditions will there be "two" solutions for the triangle?



8. Two sides of a parallelogram are 25 cm. and 35 cm., and one angle is 36°. Find the lengths of its diagonals. Draw a diagram.

9a) Given vector $v = -4i + 8j$, find the magnitude and direction of $v$.

b) Given vector $v$ with magnitude 12 and direction $\theta = 300°$, find the component form of $v$.

10. From a point A on the ground outside a building, the angle of elevation to the top of the building is 24°. From the same point, the angle of elevation to a window located 20 feet below the top of the building is 18°. Find the height of the building.

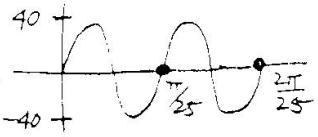
11. From city A to city B, a plane flies 650 miles at a bearing of N 48° E. From city B to city C, the plane flies 810 miles at a bearing of S 65° E. Find the distance and the bearing from A to C. [Hint: draw a figure with city A at the origin, and use vectors!]

1. $y = 40\sin 50t$

Amp = 40

Period = $\frac{2\pi}{50} = \frac{\pi}{25}$

P.S. = 0




2a) $t = 120°, 240°$

b) $t = 120°$

Not the same. $y = \arccos x$ is uniquely defined. Only one answer to a).

3a) $\sin 40°$

b) $\cos 32°$

c) $\sin^2 48°$

d) $-\cos 2(48°)$ $-\cos 96°$

e) 1

4. $LHS = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$

$= \frac{(1+\sin x)-(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \frac{1+\sin x - 1 + \sin x}{1-\sin^2 x}$

$= \frac{2\sin x}{\cos^2 x}$

$= \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= 2\tan x \sec x = RHS$

5. $\sin 3x = \sin(2x+x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= (2\sin x \cos x)\cos x + (1-2\sin^2 x)\sin x$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$.

6. $\sec^2 x + 2\sec x - 8 = 0$

$(\sec x + 4)(\sec x - 2) = 0$

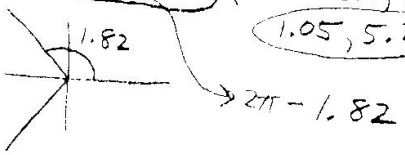
$\sec x = -4 \quad \sec x = 2$

$\cos x = -\frac{1}{4} \quad \cos x = \frac{1}{2}$

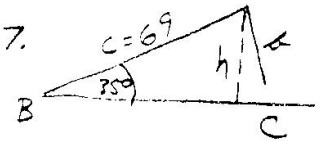
$x = 1.82$ QII $\quad x = \pi/3, 5\pi/3$

$= 4.46$ QIII $\quad (2\pi - 1.82)$

1.05, 5.24




7.




One right triangle if $a = h$

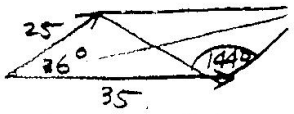
$\sin 35° = \frac{h}{69}$

$h = 69\sin 35°$

$= 39.58 = a$

Two solutions if $39.58 < a < 69$

8.




TWO WAYS: LAW of COSINE

$c^2 = a^2 + b^2 - 2ab\cos C$

$c_1^2 = 25^2 + 35^2 - 2(25)(35)\cos$.

$= 434.22$

$c_1 = 20.84$ cm

$c_2^2 = 25^2 + 35^2 - 2(25)(35)\cos 14$

$= 3265.7797$

$c_2 = 57.15$ cm

– OR –

VECTORS

$\text{norm}([35\angle 0] + [25\angle 36]) = 57.15$ cm

$\text{norm}([35\angle 0] - [25\angle 36]) = 20.84$ cm.

9a) $\vec{v} = -4\vec{i} + 8\vec{j}$

Two WAYS:

norm $[-4, 8] = 8.94$

[vector, FS (cplx), FS (angle)]

angle = 116.6°

– OR –

$c^2 = \sqrt{(-4)^2 + 8^2} = 4\sqrt{5}$

$\theta = \arctan\frac{y}{x} = \arctan\frac{8}{-4} + 180°$ (QII)

$= 116.6°$

b) $[12\angle 300]$

$= [6, -10.39]$

or $6\vec{i} - 10.39\vec{j}$

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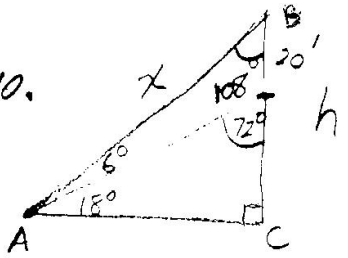
10.




$$\frac{\sin 6^\circ}{20} = \frac{\sin 108}{x}$$

$$x = \frac{20 \sin 108}{\sin 6^\circ}$$

$$= 181.97$$

$$\sin 24^\circ = \frac{h}{x}$$

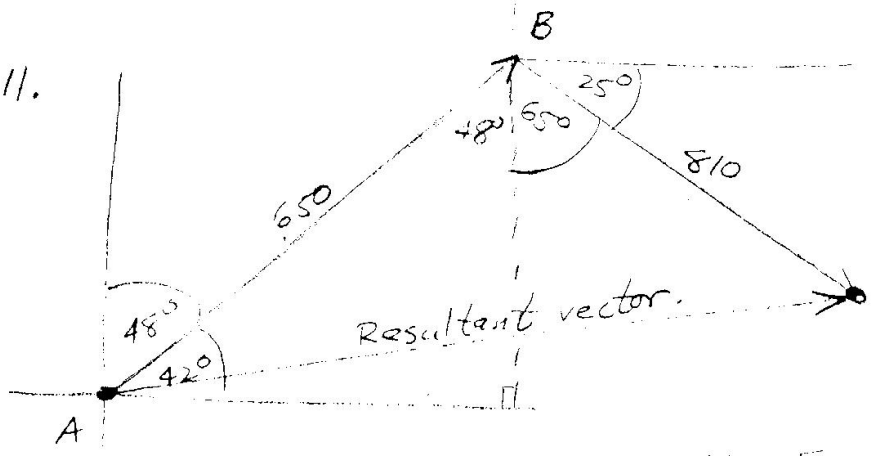
$$h = x \sin 24^\circ$$

$$= \frac{20 \sin 108 \sin 24^\circ}{\sin 6}$$

$$= 181.97 \sin 24^\circ$$

$$= 74.01'$$

11.




Resultant vector $= [650 \angle 42] + [810 \angle -25]$ or $\angle 270 + 65^\circ$ $\angle 335^\circ$

$$= [1217.15, 92.61]$$

norm = 1220.67 miles

angle = 4.35°

–OR– SAS Law of cosines.

$$c^2 = a^2 + b^2 - 2ab \cos \angle C$$

$$= 650^2 + 810^2 - 2(650)(810) \cos 113^\circ$$

$$= 1490539.575$$

$c =$ 1220.67 miles