MAC 2233 EXAM 3B **Dr. Rapalje** **NAME** _____

Sections 3.4 – 4.3

Show all work on separate paper. Turn in ALL worksheets.

1. Use implicit differentiation to find $\frac{dy}{dx}$: $y^2 = 4x + 3y - 6$.

2. Find $\frac{dy}{dx}$ evaluated at $x = 3$, $y = 2$: $x^2 + y^2 = xy + 7$.

3. The radius of a spherical balloon is increasing at the rate of 3 centimeters per minute. How fast is the volume changing when the radius is 10 centimeters? [Hint: $V = \frac{4}{3}\pi r^3$.

4. Find the value of $12,000 if it is invested for 20 years at 6% interest per year compounded:

   a) annually b) semiannually

5. Find the value of $12,000 if it is invested for 20 years at 6% interest per year compounded:

   a) daily (365 days) b) continuously

6. How much would you have to invest now at 6% interest per year compounded continuously in order to have $1,000,000 when you retire in 30 years?

7. a) $\ln e^{10x}$ b) $\log_4 \frac{1}{2}$ c) $\ln \frac{1}{\sqrt[3]{e}}$ d) $\ln x^3 - 5\ln x$

8. A bank offers 6% interest per year compounded continuously. How long will it take an investment to

   a) double b) triple

9. Given $f(x) = x^2 \ln x - x^2$, find

a) $f'(x)$ b) $f'(e)$ c) $f''(x)$ d) $f''(e)$

10. Find $f'(x)$ for each of the following:

a) $f(x) = \ln\left(\frac{x}{e^x}\right)$ b) $f(x) = x^4 e^{3x}$

11. Given $f(x) = e^{(x^2+4)}$, find

a) $f'(x)$ b) $f''(x)$

12. Manufacturing radios costs \$2 per unit produced, plus \$7000 in fixed costs. If x is the number of radios produced, then the selling price $p(x)$ of the radios is given by $p(x) = 10 - 0.001\,x$.

a) Find the cost function C(x) for producing x radios.
b) Find the revenue function R(x) from the sale of x radios.
c) Find the profit function P(x).
13. d) Find the marginal revenue and the marginal profit.
e) For what value of x will profit be maximum?
f) Find the maximum profit.

14. Given that $y = e^x$. By taking the *ln* of both sides of the equation, and by using implicit differentiation, show that $y' = e^x$.

15. A walnut grower knows that if 20 trees are planted per acre, each tree will average 60 pounds of nut per year. For each additional tree planted per acre (up to 15) the average yield per tree drops by 2 pounds. How many trees should be planted per acre to maximize the yield per acre? What is the maximum yield?

MAC 2233 Exam 3B Solutions

1. $y^2 = 4x + 3y - 6$

$2y\frac{dy}{dx} = 4 + 3\frac{dy}{dx}$

$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 4$

$\frac{dy}{dx}(2y-3) = 4$

$\frac{dy}{dx} = \frac{4}{2y-3}$

2. $x^2 + y^2 = xy + 7$

$2x + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y \cdot 1$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$

$\frac{dy}{dx}(2y-x) = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$

at $(3,2)$ $y' = \frac{2-6}{4-3} = -4$

3. $V = \frac{4}{3}\pi r^3$ $\quad \frac{dr}{dt} = 3 \text{ cm/min}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\quad r = 10 \text{ cm}$

$= 4\pi \cdot 10^2 \cdot 3 \text{ cm}^3/\text{min}$.

$= 1200\pi \text{ cm}^3/\text{min}$.

4. $A = P(1+r)^{nt}$

a) $= 12000(1+.06)^{20}$

$= 38,485.63$

b) $A = 12,000(1.03)^{40}$

$= 39,144.45$

5a) $A = 12000\left(1 + \frac{.06}{365}\right)^{20 \cdot 365}$

$= 39,837.47$

b) $A = Pe^{rt}$

$= 12000e^{(.06)(20)}$

$= 39,841.40$

$A = Pe^{rt}$

6. $1000000 = Pe^{.06 \cdot 30}$

$P = \frac{1,000,000}{e^{1.8}}$

$= 165,298.89$

7a) $\ln e^{10x} = 10x$

b) $\log_4 \frac{1}{2} = x$

$4^x = \frac{1}{2}$

$2^{2x} = 2^{-1}$

$2x = -1$

$x = -1/2$

also $\log_4 .5 = \frac{\ln .5}{\ln 4} = -.5$

c) $\ln \frac{1}{\sqrt[3]{e}}$

$\ln e^{-1/3} = -1/3$

d) $\ln x^3 - 5\ln x$

$3\ln x - 5\ln x$

$-2\ln x$

8. $A = Pe^{rt}$

a) $2P = Pe^{rt}$

$2 = e^{rt}$

$\ln 2 = \ln e^{rt}$

$\ln 2 = rt$

$t = \frac{\ln 2}{r}$

$= \frac{\ln 2}{.06} = 11.55 \text{ yrs.}$

b) $3P = Pe^{rt}$

$3 = e^{rt}$

$\ln 3 = \ln e^{rt}$

$\ln 3 = rt$

$t = \frac{\ln 3}{.06}$

$= 18.3 \text{ yrs}$

9. $f(x) = x^2 \ln x - x^2$

a) $f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x - 2x$

$= x - 2x + 2x\ln x$

$= 2x\ln x - x$

b) $f'(e) = 2e\ln e - e$

$= e$

c) $f''(x) = 2x \cdot \frac{1}{x} + \ln x \cdot 2 - 1$

$= 2 - 1 + 2\ln x$

$= 1 + 2\ln x$

d) $f''(e) = 1 + 2\ln e$

$= 3$

10a) $f(x) = \ln\left(\frac{x}{e^x}\right)$

$f(x) = \ln x - \ln e^x$

$= \ln x - x$

$f'(x) = \frac{1}{x} - 1$ or $\frac{1-x}{x}$

b) $f(x) = x^4 e^{3x}$

$f'(x) = x^4 \cdot e^{3x} \cdot 3 + e^{3x} \cdot 4x^3$

$= 3x^4 e^{3x} + 4x^3 e^{3x}$

$= x^3 e^{3x}(3x+4)$

Check: Use calculator

[2nd][calc] [der1] $x^2\ln x - x^2$, x, e^1) ENTER ANS = 2.718...

2nd ENTRY (Replace 1 in der1 with 2) ENTER ANS = 3

11. $f(x) = e^{x^2+4}$

$f'(x) = e^{x^2+4} \cdot 2x$

$f''(x) = e^{x^2+4} \cdot 2 + 2x \cdot e^{x^2+4} \cdot 2x$

$= 2e^{x^2+4} + 4x^2 e^{x^2+4}$

$= 2e^{x^2+4}(1+2x^2)$

12 a) $C(x) = 2x + 7000$

b) $R(x) = SP \times \text{Number sold}$

$= (10 - 0.001x)x = 10x - 0.001x^2$

c) $P(x) = R(x) - C(x)$

$= 10x - 0.001x^2 - 2x - 7000$

$= -0.001x^2 + 8x - 7000$

13. d) $MR = \frac{dR}{dx} = 10 - 0.002x$

$MP = \frac{dP}{dx} = -0.002x + 8$

e) Profit = Max when $MP = 0$

$-0.002x + 8 = 0$

$8 = 0.002x$

$x = \frac{8}{0.002} = 4000$ units

f) Max Profit $= -0.001x^2 + 8x - 7000$

$= -0.001(4000)^2 + 8(4000) - 7000$

$= \$9000$

14. $y = e^x$

$\ln y = \ln e^x = x$

$y \cdot \frac{1}{y}\frac{dy}{dx} = 1 \cdot y$

$\frac{dy}{dx} = y$

$\frac{dy}{dx} = e^x$

15. Let x = number of additional trees above 20.

No of trees $= 20 + x$

No of lbs per tree $= 60 - 2x$

Yield = No trees × No lbs per tree

$= (20+x)(60-2x)$

$= 1200 + 20x - 2x^2$

$\frac{dY}{dx} = 20 - 4x = 0$

$x = 5$ additional trees.

→ No trees $= 20 + x$

$= 20 + 5 = 25$

Pounds per tree $= 60 - 2x$

$= 60 - 2(5)$

$= 50$ lbs/tree

or Yield $= 25 \times 50$

$= 1250$ lbs/acre