MAC 2233 EXAM 3D **Dr. Rapalje** **NAME** _____
Sections 3.4 – 4.3

Show all work on separate paper. Turn in ALL worksheets.

1. Use implicit differentiation to find $\frac{dy}{dx}$: $x^2y = 8$.

2. A company's demand equation is $x = \sqrt{650 - p^2}$ where p is the price in dollars. Find $\frac{dp}{dx}$ when p = 5.

3. A cube of ice is melting so that each side is decreasing at the rate of 2 inches per hour. Find how fast the volume of the ice is decreasing at the moment when each edge is 10 inches long.

4. Find the value of $150,000 if it is invested for 8 years at 6% interest per year compounded:

   a) annually b) quarterly

5. Find the value of $150,000 if it is invested for 30 years at 6% interest per year compounded:

   a) daily (365 days) b) continuously

6. How much would you have to invest now at 6% interest per year compounded continuously in order to have $1,000,000 when you retire in

   a) 10 years b) 25 years?

7. A bank offers 6% interest per year compounded continuously. How long will it take an investment to

   a) double b) triple

8. Simplify the following expressions

a) $\ln\left(\frac{1}{e^3}\right)$ b) $\log_{16} 4$ c) $\ln\sqrt{e^5}$ d) $\ln x^7 - 3\ln x^3 + 2\ln x^2$

9. Given $f(x) = e^{x^3}$, find

a) $f'(x)$ b) $f''(x)$ (Give factored form!)

10. Given $f(x) = \ln(x^2 + 4)$, find

a) $f'(x)$ b) $f''(x)$

11. Find $f'(x)$ for each of the following:

a) $f(x) = x^3 e^{2x}$ b) $f(x) = \ln\left(x^3 e^{2x}\right)$

[Hint: Simplify f(x) first!]

12. Given that $y = e^x$. By taking the ***ln*** of both sides of the equation, and by using implicit differentiation, show as we did in class, that $y' = e^x$.

13. A rental car company can rent 60 cars if it charges \$80 per weekend. It estimates that for each \$5 price increase it will rent 3 fewer cars. What price should it charge to maximize its revenue? How many cars will it rent at this price?

MAC 2233 Concepts of Calculus EXAM 3D Solutions

1. $x^2y = 8$ (Product Rule!)

$x^2 \cdot \frac{dy}{dx} + y \cdot 2x = 0$

$x^2 \frac{dy}{dx} = -2xy$

$\frac{dy}{dx} = -\frac{2xy}{x^2} = \boxed{-\frac{2y}{x}}$

(See p. 258 #24)

2. $x = \sqrt{650 - p^2}$

$x^2 = 650 - p^2$

$2x = -2p \frac{dp}{dx}$

$\frac{dp}{dx} = -\frac{x}{p}$

when $p = 5$, $x = \sqrt{650 - 25} = 25$

$\frac{dp}{dx} = -\frac{25}{5} = \boxed{-5}$

(See p. 258 #38)

3. $V = x^3$; $\frac{dx}{dt} = -5$ in/hr. $x = 10$ in.

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3 \cdot 10^2 \text{in}^2 \cdot (-2 \text{ in/hr})$

$= \boxed{-600 \text{ in}^3/\text{hr}}$

(See # 58 p. 266)

4. $A = P(1+r)^n$

a) $A = 150{,}000(1+.06)^8 = \boxed{\$239{,}077.21}$

b) $A = 150{,}000\left(1+\frac{.06}{4}\right)^{32} = \boxed{\$241{,}548.65}$

5. a) $A = 150{,}000\left(1+\frac{.06}{365}\right)^{30 \cdot 365} = \boxed{\$907{,}312.89}$

b) $A = Pe^{rt}$

$A = 150000\, e^{(.06 \times 30)} = \boxed{\$907{,}447.12}$

6 a) $1000000 = Pe^{(.06 \times 10)}$

$P = \frac{1000000}{e^{.6}} = \boxed{\$548{,}811.64}$

b) $P = \frac{1000000}{e^{(.06 \times 25)}} = \boxed{\$223{,}130.16}$

7a) $2P = Pe^{rt}$

$2 = e^{.06t}$

$\ln 2 = \ln e^{.06t}$

$\ln 2 = .06t$

$t = \frac{\ln 2}{.06} \approx \boxed{11.55 \text{ yrs}}$

b) $3P = Pe^{rt}$

$3 = e^{.06t}$

$\ln 3 = \ln e^{.06t}$

$\ln 3 = .06t$

$t = \frac{\ln 3}{.06} \approx \boxed{18.31 \text{ yrs}}$

8a) $\ln\left(\frac{1}{e^3}\right) = \ln 1 - \ln e^3$

$= 0 - 3 = \boxed{-3}$

b) $\log_{16} 4 = \frac{\ln 4}{\ln 16} = \boxed{\frac{1}{2}}$

c) $\ln \sqrt{e^5} = \ln e^{5/2} = \boxed{5/2}$

d) $\ln x^7 - 3\ln x^3 + 2\ln x^2$

$7\ln x - 9\ln x + 4\ln x = \boxed{2\ln x}$

9. $f(x) = e^{x^3}$

a) $f'(x) = \boxed{e^{x^3} \cdot 3x^2}$ Product Rule!!

b) $f''(x) = e^{x^3} \cdot 6x + 3x^2 \cdot e^{x^3} \cdot 3x^2$

$= \boxed{e^{x^3}(6x + 9x^4) \text{ or } 3xe^{x^3}(2+3x^3)}$

10. $f(x) = \ln(x^3+4)$ a) $f'(x) = \frac{1}{x^3+4} \cdot 2x$

b) $f''(x) =$ [Quotient Rule!] $= \boxed{\frac{2x}{x^3+4}}$

$= \frac{(x^3+4)(2) - 2x(2x)}{(x^3+4)^2}$

$= \frac{2x^3+8-4x^2}{(x^3+4)^2} = \boxed{\frac{-2x^2+8}{(x^3+4)^2} \text{ or } \frac{2(4-x^2)}{(x^3+4)^2}}$

11a) $f(x) = x^3e^{2x}$ (PROD RULE!)

$f'(x) = x^3 \cdot e^{2x} \cdot 2 + e^{2x} \cdot 3x^2$

$= \boxed{x^2e^{2x}(2x+3)}$

b) $f(x) = \ln(x^3e^{2x})$

$f(x) = \ln x^3 + \ln e^{2x}$

$f(x) = 3\ln x + 2x$

$f'(x) = 3 \cdot \frac{1}{x} + 2$

$= \frac{3}{x} + \frac{2x}{x}$

$= \boxed{\frac{2x+3}{x}}$

12. $y = e^x$

$\ln y = \ln e^x$

$\ln y = x$

$\frac{1}{y}\frac{dy}{dx} = 1$

$y \cdot \frac{1}{y}\frac{dy}{dx} = y \cdot 1$

$\frac{dy}{dx} = y = e^x$

13. Let x = no. price increases.

Price = $80 + 5x$

Number cars rented = $60 - 3x$

REVENUE = $(80+5x)(60-3x)$

$R(x) = 4800 - 240x + 300x - 15x^2$

$R(x) = 4800 + 60x - 15x^2$

$R'(x) = 60 - 30x = 0$ at MAX

$60 = 30x$

$x = 2$ increases.

$P = 80 + 5x = \boxed{\$90}$

$N = 60 - 3x = \boxed{54 \text{ cars}}$