MAC 2233 EXAM 4C **Dr. Rapalje** **NAME** ______________________

Chapter 5

Show all work on separate paper. Turn in ALL worksheets.

1. $\int (4\sqrt{x} - 5)\, dx$

2. $\int \frac{1}{x^2}\, dx$

3. $\int \left(e^{4x} - \frac{4}{e^{4x}}\right) dx$

4. $\int \left(\frac{x^2 + 8x - 3}{x}\right) dx$

5. $\int \frac{dx}{5x}$

6. $\int 24e^{-\frac{3x}{2}}\, dx$

7. Find the value of the integral by calculus. Check by calculator
$\int_0^1 \left(x^{99} + x^9 + 1\right) dx$

8. Given $\int_0^1 (12e^{3x})\, dx$
a) Find the exact value using calculus.
b) Find the decimal approximation (using the calculator!)

9. Find the area under the curve $f(x) = 3x^2 - 4x + 5$ from $x = 2$ to $x = 5$.

10. Find the area between the curves $y = 12x - 3x^2$ and $y = 6x - 24$.

11. Find the average value of the function $f(x) = \sqrt{x}$ on [0, 16].

In 12 – 15, find each integral.

12. $\int (x^3+6)^4\, x^2 dx$ 13. $\int \frac{x^2\, dx}{\sqrt{x^3 + 6}}$ 14. $\int \frac{x^2}{x^3 + 6}\, dx$ 15. $\int \frac{(\ln x)^2}{x}\, dx$

16. Evaluate: $\int_0^3 x^2\sqrt{x^3 + 9}\, dx$ a) using calculus b) using calculator.

17. World copper consumption is running at the <u>rate</u> of $20\, e^{0.05t}$ million tons of copper per year, beginning at t=0 in year 2000.
a) Find a formula for the total number of tons used after 2000.
(Be sure to find the constant c).
b) How many years will it take to use up 800 million tons of copper?

MAC 2233 Exam 4C Solutions

1. $\int(4x^{1/2}-5)\,dx$

$= 4\frac{x^{3/2}}{3/2} - 5x + c$

$= \frac{8}{3}x^{3/2} - 5x + c$

2. $\int x^{-2}\,dx$

$= \frac{x^{-1}}{-1} + c$

$= -\frac{1}{x} + c$

3. $\int(e^{4x} - 4e^{-4x})\,dx$

$\frac{1}{4}e^{4x} - 4\cdot\frac{e^{-4x}}{-4} + c$

$\frac{1}{4}e^{4x} + e^{-4x} + c$

4. $\int\left(\frac{x^2+8x-3}{x}\right)dx$

$= \int(x+8-3x^{-1})\,dx$

$= \frac{x^2}{2} + 8x - 3\ln x + c$

5. $\int\frac{dx}{5x}$

$= \frac{1}{5}\int\frac{dx}{x}$

$= \frac{1}{5}\ln x + c$

6. $\int 24e^{-\frac{3x}{2}}\,dx$

$= \cancel{24}^{8}\cdot -\frac{2}{\cancel{3}}e^{-\frac{3x}{2}} + c$

$= -16e^{-\frac{3x}{2}} + c$

7. $\int_0^1(x^{99}+x^9+1)\,dx$

$= \frac{x^{100}}{100} + \frac{x^{10}}{10} + x\Big|_0^1$

$= \left(\frac{1}{100} + \frac{1}{10} + 1\right) - (0+0+0)$

$= .01 + .1 + 1 = 1.11$

8 a) $\int_0^1(12e^{3x})\,dx$

$\cancel{12}^{4}\frac{e^{3x}}{\cancel{3}}\Big|_0^1$

$= 4e^{3x}\Big|_0^1$

$= 4e^3 - 4\cdot 1$

b) fnInt$(12e^{(3x)}, x, 0, 1)$

$= 76.34$

9. $\int_2^5(3x^2-4x+5)\,dx$

$= x^3 - 2x^2 + 5x\Big|_2^5$

$= (125-50+25) - (8-8+10)$

$= 100 - 10 = 90$

fnInt$(3x^2-4x+5, x, 2, 5)$

$= 90$

10. $y = 12x - 3x^2 =$ upper

$y = 6x - 24 =$ Lower

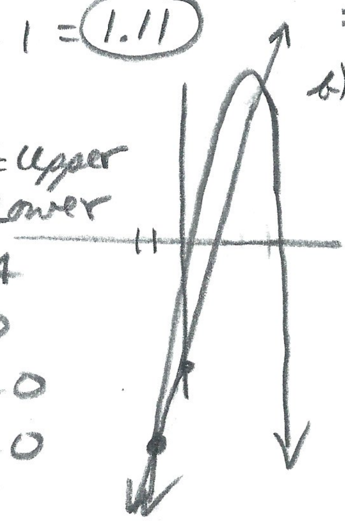
$12x - 3x^2 = 6x - 24$

$-3x^2 + 6x + 24 = 0$

$-3(x^2 - 2x - 8) = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad x = -2$



$A = \int_{-2}^{4}(\text{upper} - \text{lower})\,dx$

$= \int_{-2}^{4}[(12x - 3x^2) - (6x-24)]\,dx$

$= \int_{-2}^{4}(12x - 3x^2 - 6x + 24)\,dx$

$= \int_{-2}^{4}(-3x^2 + 6x + 24)\,dx$

fnInt$(-3x^2+6x+24, x, -2, 4)$

$= 108$

11. $f_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx \qquad f(x) = \sqrt{x}$ on $[0, 16]$

$= \frac{1}{16-0}\int_0^{16}\sqrt{x}\,dx$

$= \frac{1}{16}\frac{2}{3}x^{3/2}\Big|_0^{16} = \frac{1}{24}\cdot 16^{3/2}$

$= \frac{1}{24}\cdot 64 = \frac{8}{3}$

12. $\int(x^3+6)^4x^2\,dx \qquad$ let $u = x^3 + 6$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \int u^4\frac{du}{3}$

$= \frac{1}{3}\frac{u^5}{5} + c$

$= \frac{1}{15}(x^3+6)^5 + c$

13. $\int \frac{x^2\,dx}{\sqrt{x^3+6}}$ Let $u = x^3+6$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \int \frac{\frac{du}{3}}{\sqrt{u}}$

$= \frac{1}{3}\int u^{-1/2}\,du$

$= \frac{1}{3}\cdot\frac{2}{1}u^{1/2} + C$

$= \frac{2}{3}\sqrt{x^3+6} + C$

14. $\int \frac{x^2}{x^3+6}\,dx$ Let $u = x^3+6$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \int \frac{\frac{du}{3}}{u}$

$= \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln u + C$

$= \frac{1}{3}\ln|x^3+6| + C$

15. $\int \frac{(\ln x)^2}{x}\,dx$ Let $u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int u^2\,du$

$= \frac{u^3}{3} + C = \frac{(\ln x)^3}{3} + C$

16. $\int_0^3 x^2\sqrt{x^3+9}\,dx$

$= \int \sqrt{x^3+9}\;x^2\,dx$ Let $u = x^3+9$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \int u^{1/2}\,\frac{du}{3}$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}(x^3+9)^{3/2}\Big|_0^3$

$= \frac{2}{9}\left[(36)^{3/2} - (9)^{3/2}\right]$

$= \frac{2}{9}\left[(216 - 27)\right]$

$\frac{2}{\cancel{9}}\cdot \cancel{189}^{\,21} = 42$

—OR—

$\text{fnInt}(x^2\sqrt{x^3+9}, x, 0, 3)$

$= 42$

17. $\frac{dC}{dt} = 20e^{0.05t}$

$C = \int 20e^{0.05t}\,dt$

$= 20\,\frac{e^{0.05t}}{.05} + C$

$= 400e^{0.05t} + C$

$C(0) = 0$, so

$0 = 400 + C \Rightarrow C = -400$

$C(t) = 400e^{0.05t} - 400$

Find t when $C = 800$

$800 = 400e^{0.05t} - 400$

$\frac{1200}{400} = \frac{400e^{0.05t}}{400}$

$3 = e^{0.05t}$

$\ln 3 = \ln e^{0.05t}$

$\ln 3 = .05t$

$t = \frac{\ln 3}{.05} \approx 21.97$ yrs.