MAC 3233 CONCEPTS of CALCULUS EXAM 1 — UCF PRACTICE EXAMS

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Problems 1–23, 3 each (right or wrong).
Part II problems, 8 each (partial credit)

In 1–2, give the domain:

1. $f(x) = \frac{4}{x^2 - 9x}$

2. $g(x) = \sqrt{x+4}$

In 3–5, find the function values: (Simplify if possible)

3. $h(x) = x^2 + 3x$

$h(-2) =$ _____

4. $f(x) = \frac{2x}{x^2+4}$

$f(3x) =$ _____

5. $g(x) = \begin{cases} 3x+1 & \text{if } x > 2 \\ -3 & \text{if } x \leq 2 \end{cases}$

$g(-2) =$ _____

In 6–7, let $f(x) = x^2 + x$, $g(x) = \frac{3x-2}{x}$. (Simplify completely including common denom. if necessary!)

6. $(f \circ g)(x) =$ _____

7. $(g \circ f)(x) =$ _____

8. Find the slope and the equation of the line through $(-1, 9)$ and $(1, 5)$. (Give eq. in form $y = mx + b$.)

In 9–10, give the limits:

9. $\lim_{x \to -1} (x^3 - 3x^2) =$

10. $\lim_{x \to 3} \frac{x-3}{x^2-9}$

11. Find all points of discontinuity: $f(x) = \frac{x^2-4}{x^2-4x}$

12. $y = \frac{1}{2x^3}$

$y' =$ _____.

13. $q = 3p^3 - 4p^2 + 6p - 1$

$\frac{dq}{dp} =$ _____

14. $D_x(\sqrt{x}) =$ _____.

15. If $f(x) = 2x(x^2 - 5x + 2)$, find the slope of the tangent line at $x = 2$.

16. Find the derivative: $f(x) = x^3(2x^4)^2$

17. $y = x^2(x-2)(x+3)$

$y' =$ _____.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____

18. Find the slope to the curve $y = \frac{x^3}{x^4+1}$ at $(1, \frac{1}{2})$.

19. $f(x) = (x^2+3)^5$

$f'(x) =$ _____.

20. $y = 2x^2 - 3x + \frac{1}{x}$.

Find y'''

21. If $r = q(20 - .1q)$ is a total revenue function, find the marginal revenue function.

22. If $C = 6 + \frac{3}{4}I - \frac{\sqrt{I}}{3}$, find the marginal propensity to consume when $I = 25$. (C is a consumption function)

23. If $c = 500 + 6q$ is a cost function, find the marginal cost when $q = 36$.

B. _____

19. _____

20. _____

21. _____

22. _____

23. _____

SHOW ALL WORK – PARTIAL CREDIT

1. Find the equation of the tangent line to the curve $y = 4x^2 + 5x + 2$ at $x = -1$.

2. If $y = (x^2-2)^5(3x+5)^4$, find y' and factor completely

3. $y = \left(\frac{2x+5}{3x+1}\right)^4$

Find $\frac{dy}{dx}$ and simplify.

4. m employees produce q units per day where $q = m(m-6)$. The demand eq. for the product is $p = 2\sqrt{q}$. Find the marginal revenue product $\frac{dr}{dm}$ when $m = 8$.

1. D: all $x \neq 0, 9$

2. D: all $x \geq -4$

3. $h(x) = x^2 + 3x$
$h(-2) = 4 - 6 = \boxed{-2}$

4. $f(x) = \dfrac{2x}{x^2+4}$
$f(3x) = \dfrac{2(3x)}{(3x)^2+4}$
$= \boxed{\dfrac{6x}{9x^2+4}}$

5. $g(-2) = \boxed{-3}$

6. $f[x] = x^2 + x$
$f[g(x)] = \left(\dfrac{3x-2}{x}\right)^2 + \left(\dfrac{3x-2}{x}\right)$
$= \dfrac{9x^2-12x+4}{x^2} + \dfrac{3x^2-2x}{x^2}$
$= \boxed{\dfrac{12x^2-14x+4}{x^2}}$

7. $g(x) = \dfrac{3x-2}{x}$
$g[f(x)] = \dfrac{3(x^2+x)-2}{x^2+x}$
$= \boxed{\dfrac{3x^2+3x-2}{x^2+x}}$

8. $(-1,9)$ & $(1,5)$
$m = \dfrac{9-5}{-1-1} = \dfrac{4}{-2} = \boxed{-2}$
$y - y_1 = m(x - x_1)$
$y - 5 = -2(x-1)$
$\boxed{y = -2x + 7}$

9. $\lim_{x \to -1} (x^3 - 3x^2) = -1 - 3$
$= \boxed{-4}$

10. $\lim_{x \to 3} \dfrac{x-3}{(x-3)(x+3)} = \boxed{\dfrac{1}{6}}$

11. $f(x) = \dfrac{x^2-4}{x^2-4x} = \dfrac{x^2-4}{x(x-4)}$
Discontin. at $\boxed{x=0, x=4}$

12. $y = \dfrac{1}{2x^3} = \dfrac{1}{2}x^{-3}$
$y' = -\dfrac{3}{2}x^{-4} = \boxed{-\dfrac{3}{2x^4}}$

13. $g = 3p^3 - 4p^2 + 6p - 1$
$\boxed{\dfrac{dg}{dp} = 9p^2 - 8p + 6}$

14. $D_x(\sqrt{x}) = D_x(x^{1/2})$
$= \dfrac{1}{2}x^{-1/2} = \boxed{\dfrac{1}{2\sqrt{x}}}$

15. $f(x) = 2x(x^2 - 5x + 2)$
$= 2x^3 - 10x^2 + 4x$
$f'(x) = 6x^2 - 20x + 4$
$f'(2) = 24 - 40 + 4$
$\boxed{-12}$

16. $f(x) = x^3(2x^4)^2$
$= x^3(4x^8)$
$= 4x^{11}$
$\boxed{f'(x) = 44x^{10}}$

17. $y = x^2(x-2)(x+3)$
$= x^2(x^2 + x - 6)$
$= x^4 + x^3 - 6x^2$
$\boxed{y' = 4x^3 + 3x^2 - 12x}$
or $\boxed{x(4x^2 + 3x - 12)}$

18. $y = \dfrac{x^3}{x^4+1}$ at $x = 1$
$y' = \dfrac{(x^4+1)3x^2 - x^3(4x^3)}{(x^4+1)^2}$
$= \dfrac{6-4}{4} = \boxed{\dfrac{1}{2}}$

19. $f(x) = (x^2+3)^5$
$f'(x) = 5(x^2+3)^4 \cdot 2x$
$\boxed{= 10x(x^2+3)^4}$

20. $y = 2x^2 - 3x + x^{-1}$
$y' = 4x - 3 - x^{-2}$
$y'' = 4 + 2x^{-3}$
$y''' = -6x^{-4} = \boxed{\dfrac{-6}{x^4}}$

21. $r = g(20 - .1g)$
$= 20g - .1g^2$
$\boxed{\dfrac{dr}{dg} = 20 - .2g}$

22. $C = 6 + \dfrac{3}{4}I - \dfrac{\sqrt{I}}{3}$, $I = 25$
$\dfrac{dC}{dI} = \dfrac{3}{4} - \dfrac{1}{6}I^{-1/2}$
$= \dfrac{3}{4} - \dfrac{1}{6} \cdot \dfrac{1}{5}$
$= \dfrac{45}{60} - \dfrac{2}{60} = \boxed{\dfrac{43}{60}}$

23. $C = 500 + 6g$.
$\boxed{\dfrac{dC}{dg} = +6}$ for all g.

1. $y = 4x^2 + 5x + 2$ $\quad x_0 = -1$, $y_0 = 4 - 5 + 2 = 1$
$y' = 8x + 5$ $\quad m = -8 + 5 = -3$
$y - 1 = -3(x+1)$
$\boxed{y = -3x - 2}$

2. $y = (x^2-2)^5(3x+5)^4$
$y' = (x^2-2)^5 4(3x+5)^3 \cdot 3$
$+ (3x+5)^4 5(x^2-2)^4 2x$
$= 2(x^2-2)^4(3x+5)^3[6(x^2-2) + 5x(3x+5)]$
$\boxed{2(x^2-2)^4(3x+5)^3(21x^2+25x-12)}$

3. $y = \left(\dfrac{2x+5}{3x+1}\right)^4$
$y' = 4\left(\dfrac{2x+5}{3x+1}\right)^3 \dfrac{(3x+1)\cdot 2 - (2x+5)\cdot 3}{(3x+1)^2}$
$= 4\dfrac{(2x+5)^3}{(3x+1)^3} \dfrac{6x+2-6x-15}{(3x+1)^2}$
$\boxed{\dfrac{-52(2x+5)^3}{(3x+1)^5}}$

4. $q = m(m-6)$ $\quad p = 2\sqrt{q}$
$\dfrac{dq}{dm} = 2m - 6$ $\quad \dfrac{dp}{dq} = q^{-1/2}$
If $m = 8$, then $q = 8 \cdot 2 = 16$
$\dfrac{dq}{dm} = 16 - 6 = 10$ $\quad p = 2\sqrt{16} = 8$
$\dfrac{dp}{dq} = 16^{-1/2} = \boxed{\dfrac{1}{4}}$
$r = pq$
$\dfrac{dr}{dm} = p\dfrac{dq}{dm} + q\dfrac{dp}{dm}$
$= p\dfrac{dq}{dm} + q\dfrac{dp}{dq} \cdot \dfrac{dq}{dm}$
$= 8 \cdot 10 + 16 \cdot \dfrac{1}{4} \cdot 10$
$= 80 + 40 = \boxed{120}$