

Show all work on this test or on separate paper as necessary. CALCULATORS ARE ALLOWED. Answers in terms of "ln" or "e" are acceptable.

In 1-7, solve for the unknown:

1. $\log_2 8 = x$

2. $\log_3 x = -2$

3. $\log_7 \sqrt{7} = x$

ea)

4. $\ln x = 4$

5. $\log .01 = x$

6. $e^{\ln(4x)} = 7$

ea)

7. $e^{4 \ln x} = 16$

Express in terms of $\ln x$, $\ln(x+1)$, and/or $\ln(x+2)$:

8. $\ln\left(\frac{x^3(x+1)^4}{\sqrt{x+2}}\right)$

9. $\ln \frac{x+2}{x^2 \sqrt{x+1}}$

ea)

10. $y = \ln\left(\frac{x^2}{2x-3}\right)$

Find y' . (Laws of logs first!)

11. $y = x^2 e^{3x}$

Find y' . (Factor for.)

12. $y = \ln(x + \sqrt{1+x^2})$

Find y' .

6ea)

13. $\int (x+1)e^{x^2+2x} dx$

14. $\int (e^{3x} - e^{-2x}) dx$

(5 ea)

15. $\int \frac{x^5 dx}{(x^6+4)}$

16. $\int \frac{x^5 dx}{(x^6+4)^2}$

17. $\int_1^e (x^{-1} + x^{-2}) dx$

(5 ea)

18. $f(x,y) = x^5 y^4 - 3x^4 y^3 + 7x^3$

$f_x =$ _____

$f_{xx} =$ _____

$f_{xy} =$ _____

(12) $f_y =$ _____

$f_{yy} =$ _____

$f_{yx} =$ _____

19. $f(x,y) = x^3 e^{2x+3y}$

Find f_x and f_y (Factor answer if neces)

$f_x =$

(8)

$f_y =$

20. For a manufacturer's product, the demand function is $q = 1000e^{-.2p}$. Find the value of p for which the revenue is maximized.

(6)

21. If $\frac{dc}{dq}$ is a marginal cost function, $\frac{dc}{dq} = \frac{5}{q+1}$, find the cost equation. If fixed costs are 30, then solve for the constant in part a)

(6)

22. If $y = x^2 \ln x$,
- give domain _____.
 - find all possible extrema.
 - for what values of x is the graph increasing?

(6)

FORMULAS:

- $\frac{d}{dx}(e^x) = e^x$ $\int e^x dx = e^x + c$
- $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$ $\int e^u du = e^u + c$
- $\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$ $\int a^u du = \frac{a^u}{\ln a} + c$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\int \frac{1}{x} dx = \ln|x| + c$
- $\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$ $\int \frac{1}{u} du = \ln|u| + c$
- $\frac{d}{dx}(\log_x u) = \frac{1}{u} \log_x e \frac{du}{dx}$

1. $\log_2 8 = x$
 $2^x = 8$
 $x = 3$

2. $\log_3 x = -2$
 $3^{-2} = x$
 $x = \frac{1}{9}$

3. $\log_7 \sqrt{7} = x$
 $7^x = \sqrt{7} = 7^{1/2}$
 $x = 1/2$

4. $\ln x = 4$
 $\log_e x = 4$
 $e^4 = x$

5. $\log_{10} .01 = x$
 $10^x = .01$
 $x = -2$

6. $e^{\ln(4x)} = 7$
 $4x = 7$
 $x = 7/4$

7. $e^{4 \ln x} = 16$
 $e^{\ln x^4} = 16$
 $x^4 = 16$
 $x = 2$

8. $\ln \frac{x^3(x+1)^4}{\sqrt{x+2}}$
 $= \ln x^3 + \ln(x+1)^4 - \ln(x+2)^{1/2}$
 $= 3 \ln x + 4 \ln(x+1) - \frac{1}{2} \ln(x+2)$

9. $\ln \frac{x+2}{x^2 \sqrt{x+1}}$
 $= \ln(x+2) - \ln x^2 - \ln(x+1)$
 $= \ln(x+2) - 2 \ln x - \ln(x+1)$

10. $y = \ln \frac{x^2}{2x-3}$
 $y = \ln x^2 - \ln(2x-3)$
 $= 2 \ln x - \ln(2x-3)$
 $y' = \frac{2}{x} - \frac{1}{2x-3} \cdot 2$
 $= \frac{2}{x} - \frac{2}{2x-3}$

11. $y = x^2 e^{3x}$
 $y' = x^2 e^{3x} \cdot 3 + e^{3x} \cdot 2x$
 $= x e^{3x} (3x+2)$

12. $y = \ln(x + \sqrt{1+x^2}) \neq \ln x + \ln \sqrt{1+x^2}$
 $y' = \frac{1}{x + \sqrt{1+x^2}} \cdot (1 + \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x)$
 $= \frac{1}{x + \sqrt{1+x^2}} \left(1 + \frac{x}{\sqrt{1+x^2}}\right)$
 $= \frac{1}{(x + \sqrt{1+x^2}) \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}} = \frac{1}{\sqrt{1+x^2}}$

13. $\int (x+1) e^{(x^2+2x)} dx$
 $= \int e^u \frac{du}{2}$ let $u = x^2+2x$
 $du = (2x+2) dx$
 $\frac{du}{2} = (x+1) dx$
 $= \frac{1}{2} e^u + c$
 $= \frac{1}{2} e^{(x^2+2x)} + c$

14. $\int (e^{3x} - e^{-2x}) dx$
 $= \frac{1}{3} e^{3x} - \frac{1}{-2} e^{-2x} + c$
 $= \frac{1}{3} e^{3x} + \frac{1}{2} e^{-2x} + c$

15. $\int \frac{x^5 dx}{x^6+4}$ let $u = x^6+4$
 $du = 6x^5 dx$
 $\frac{du}{6} = x^5 dx$
 $= \frac{1}{6} \int \frac{du}{u}$
 $= \frac{1}{6} \ln u + c$
 $= \frac{1}{6} \ln(x^6+4) + c$

16. $\int \frac{x^5 dx}{(x^6+4)^2}$ let $u = x^6+4$
 $du = 6x^5 dx$
 $= \frac{1}{6} \int \frac{du}{u^2} = \frac{1}{6} \int u^{-2} du$
 $= \frac{1}{6} \frac{u^{-1}}{-1} + c = -\frac{1}{6} \frac{1}{(x^6+4)} + c$

17. $\int_1^e (x^{-1} + x^{-2}) dx$
 $= \ln x + \frac{x^{-1}}{-1} \Big|_1^e$
 $= (\ln e - \frac{1}{e}) - (\ln 1 - 1)$
 $= 1 - \frac{1}{e} - 0 + 1$
 $= 2 - \frac{1}{e}$

18. $f(x,y) = x^5 y^4 - 3x^4 y^3 + 7x^3$
 $f_x = 5x^4 y^4 - 12x^3 y^3 + 21x^2$
 $f_{xx} = 20x^3 y^4 - 36x^2 y^3 + 42x$
 $f_{xy} = 20x^4 y^3 - 36x^3 y^2$
 $f_y = 4x^5 y^3 - 9x^4 y^2$
 $f_{yy} = 12x^5 y^2 - 18x^4 y$
 $f_{yx} = 20x^4 y^3 - 36x^3 y^2$

19. $f(x,y) = x^3 e^{2x+3y}$
 $f_x = x^3 e^{2x+3y} \cdot 2 + e^{2x+3y} \cdot 3x^2$
 $= x^2 e^{2x+3y} (2x+3)$
 $f_y = 3x^3 e^{2x+3y}$

20. $r = pg$
 $= 1000p e^{-2p}$
 $\frac{dr}{dp} = 1000 p e^{-2p} (-2) + e^{-2p} (1000)$
 $= e^{-2p} (-2000p + 1000) = 0$
 $e^{-2p} \neq 0$ $p = 5$

21. $\text{cost} = \int \frac{5}{g+1} dg$
 $\text{cost} = 5 \ln(g+1) + c$
 $g = 0, \text{cost} = 30$
 $30 = 5 \ln(1) + c$
 $c = 30$

22. $y = x^2 \ln x$ a) $D: x > 0$
 $y' = x^2 \cdot \frac{1}{x} + \ln x (2x)$
 $= x + 2x \ln x = 0$
 $x(1 + 2 \ln x) = 0$
 $x = 0$ $1 + 2 \ln x = 0$
 $2 \ln x = -1$
 $\ln x = -1/2$
 $x = e^{-1/2}$
 However, $x \neq 0$.
 b) $x = e^{-1/2} \approx .607$

22c) $x > e^{-1/2}$
 $x > .607$