CALCULUS I EXAM 2 UCF

Dr. Rapalje

Calculators allowed, but not required. Show all work to justify answers algebraically. SIMPLIFY ANSWERS COMPLETELY! SHOW WORK!

1. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{x^2}$.

2. A rocket is propelled upward so that in t seconds it reaches a height of $s = 5t^3$ ft.
   a) How high does the rocket travel in the first 3 seconds of flight?
   b) What is the average velocity during the first 3 seconds?
   c) What is the instantaneous velocity at the end of 3 seconds?

3. If $f(-2) = -2$, $f'(-2) = -5$, $g(-2) = 1$, $g'(-2) = 7$, find:
   a) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\Bigg|_{x=-2}$ b) $\frac{d}{dx}\left[g[f(x)]\right]\Bigg|_{x=-2}$

4. Find $f'(x)$ if $f(x) = 3x^{10} + 7x^2 - 3x - 5 + \frac{1}{2x} + \sqrt{3x-2}$
   (Do not simplify!)

5. For $f(x) = \sqrt{3x+1}\,(x-1)^2$, find $f'(x)$, and find all values of x for which $f'(x) = 0$.

6. Find $\frac{d}{dx}\left[\left(\frac{x-5}{2x+1}\right)^3\right]$ and simplify completely.

7. Find $\frac{d^2y}{dx^2}$ if $y = x^2 \sin(3x)$.

8. Use the limit definition of the derivative to show that if $f(x) = \sin x$, then $f'(x) = \cos x$.

9. Use the quotient rule to show that if $f(x) = \cot x$, then $f'(x) = -\csc^2 x$.

10. Find: a) $\frac{d}{dx}(\sin x^2)$ b) $\frac{d}{dx}(\sin^2 x)$.

11. Find: $\frac{d}{dx}\left[\sec^3(9x+1)\right]$.

12. Find $\frac{dy}{dx}$ for $x^3y^2 - 5x^2y + 5x = 10$

13. Find all values of x such that the tangent line to $f(x) = 2x^3 - x^2$ is perpendicular to $x + 4y = 10$.

14. Find dy and Δy for $y = x^2 - 3x$

15. Use a Δx approximation (i.e., $f(x+\Delta x) \approx f(x) + f'(x)\cdot\Delta x$) to estimate $\sqrt{80.8}$.

CALCULUS I Exam 2 Solutions.

Dr. Rapalje

1. $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$, $f(x)=\frac{1}{x^2}$

$=\lim\limits_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

$=\lim\limits_{h\to 0}\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\cdot\frac{1}{h}$

$=\lim\limits_{h\to 0}\frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$

$=\lim\limits_{h\to 0}\frac{\cancel{h}(-2x-h)}{x^2(x+h)^2}\cdot\frac{1}{\cancel{h}}$

$=\frac{-2x}{x^2\cdot x^2}=\frac{-2}{x^3}$

2. $s=5t^3$

a) $s=5\cdot 3^3=135$ ft

b) $V_{av.}=\frac{dist}{t}=\frac{135}{3}$

$=45$ ft/sec

c) $\frac{ds}{dt}=15t^2$

$=15\cdot 9=135$ ft/sec

3a) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)\cdot g'(x)}{[g(x)]^2}$

$=\frac{1\cdot(-5)-(-2)(7)}{1}$

$=-5+14=9$

3b) $\frac{d}{dx}\left[g[f(x)]\right]=g'[f(x)]\cdot f'(x)$

$=g'(-2)\cdot f'(-2)$

$=7\cdot(-5)=-35$

4. $f(x)=3x^{10}+7x^2-3x-5+\frac{1}{2}x^{-1}+(3x-2)^{1/2}$

$f'(x)=30x^9+14x-3-\frac{1}{2}x^{-2}+\frac{1}{2}(3x-2)^{-1/2}\cdot 3$

5. $f(x)=\sqrt{3x+1}\,(x-1)^2$

$f'(x)=\sqrt{3x+1}\;2(x-1)+(x-1)^2\frac{1}{2}(3x+1)^{-1/2}\cdot 3$

$=(x-1)(3x+1)^{-1/2}\left[2(3x+1)+\frac{3}{2}(x-1)\right]$

$=\frac{(x-1)}{\sqrt{3x+1}}\left(\frac{12x+4+3x-3}{2}\right)$

$=\frac{(x-1)(15x+1)}{2\sqrt{3x+1}}=0$ at $x=1,-1/15$

6. $\frac{d}{dx}\left(\frac{x-5}{2x+1}\right)^3=3\left(\frac{x-5}{2x+1}\right)^2\left(\frac{(2x+1)\cdot 1-(x-5)\cdot 2}{(2x+1)^2}\right)$

$=\frac{3(x-5)^2}{(2x+1)^2}\,\frac{2x+1-2x+10}{(2x+1)^2}$

$=\frac{3(x-5)^2(11)}{(2x+1)^4}=\frac{33(x-5)^2}{(2x+1)^4}$

7. $y=x^2\sin 3x$

$\frac{dy}{dx}=x^2\cdot\cos 3x\cdot 3+\sin 3x\cdot 2x$

$=3x^2\cos 3x+2x\sin 3x$

$\frac{d^2y}{dx^2}=3x^2(-\sin 3x\cdot 3)+(\cos 3x)6x+2x\cos 3x\cdot 3+\sin 3x\cdot 2$

$=-9x^2\sin 3x+6x\cos 3x+6x\cos 6x+2\sin 3x$

$=-9x^2\sin 3x+12x\cos 3x+2\sin 3x$

8. $f(x)=\sin x$ $\quad f(x+h)=\sin(x+h)$

$f'(x)=\lim\limits_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$=\lim\limits_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

$=\lim\limits_{h\to 0}\left[\frac{\sin x(\cos h-1)}{h}+\frac{\cos x\sin h}{h}\right]$

$=\sin x\lim\limits_{h\to 0}\left(\frac{\cos h-1}{h}\right)+\cos x\lim\limits_{h\to 0}\frac{\sin h}{h}$

$=\sin x\cdot 0+\cos x\cdot 1=\cos x$

9. $f(x)=\cot x=\frac{\cos x}{\sin x}$

$f'(x)=\frac{\sin x(-\sin x)-\cos x\cos x}{\sin^2 x}$

$=\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}$

$=-\frac{1}{\sin^2 x}=-\csc^2 x$

10a) $\frac{d}{dx}(\sin x^2)$

$=\cos(x^2)\cdot 2x$

$=2x\cos(x^2)$

b) $\frac{d}{dx}(\sin^2 x)$

$=2\sin x\cos x$

11. $\frac{d}{dx}\left[\sec^3(9x+1)\right]=3\sec^2(9x+1)\cdot\left[\sec(9x+1)\tan(9x+1)\cdot 9\right]$

$=27\sec^3(9x+1)\tan(9x+1)$

12. $x^3y^2-5x^2y+5x=10$

$x^3 2y\frac{dy}{dx}+y^2 3x^2-5x^2\frac{dy}{dx}+y(-10x)+5=0$

$(2x^3y-5x^2)\frac{dy}{dx}=10xy-3x^2y^2-5$

$\frac{dy}{dx}=\frac{10xy-3x^2y^2-5}{2x^3y-5x^2}$

13. $f(x)=2x^3-x^2$

$f'(x)=6x^2-2x=$ slope

$6x^2-2x=4$

$6x^2-2x-4=0$

$2(3x^2-x-2)=0$

$2(3x+2)(x-1)=0$

$x=-2/3 \quad x=1$

$x+4y=10$

$y=-\frac{1}{4}x+\frac{10}{4}$

$m=-1/4$

$m_\perp=4$

14. $y=x^2-3x$

$dy=(2x-3)dx$

$\Delta y=y(x+\Delta x)-y(x)$

$=(x+\Delta x)^2-3(x+\Delta x)-(x^2-3x)$

$=\cancel{x^2}+2x\Delta x+\Delta x^2-\cancel{3x}-3\Delta x-\cancel{x^2}+\cancel{3x}$

$=(2x+\Delta x-3)\Delta x$

15. $f(x)=\sqrt{x}$ $\quad f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$x+\Delta x=80.8$

$x=81,\ \Delta x=-.2$

$f(x+\Delta x)=f(x)+f'(x)\Delta x$

$=9+\frac{1}{18}(-.2)$

$=9-\frac{1}{90}$