CALCULUS I EXAM 4 (Chapter 5 Anton) UCF Dr. RAPALJE

Show all work on separate paper.

1. $\int x(1+x^3)\,dx$ 2. $\int \frac{\sin x}{\cos^2 x}\,dx$ 3. $\int \frac{3x\,dx}{\sqrt{4x^2+5}}$

4. $\int x^2 \sec^2(x^3)\,dx$ 5. $\int_1^3 \left(x^2 + \frac{1}{x^2}\right)dx$ 6. $\int_4^9 2y\sqrt{y}\,dy$

7. $\int_0^8 x\sqrt{1+x}\,dx$ 8. $\int_0^{\pi/4} \tan^3 x \sec^2 x\,dx$

9. Express in Σ notation: $1+3+5+7+\ldots+23$

10. Use summation formula to compute: $\sum_{k=50}^{100} k^2$

In 11-12, graph each function and use formulas from geometry to evaluate:

11. $\int_{-2}^{2} |2x-3|\,dx$ 12. $\int_0^3 \sqrt{9-x^2}\,dx$

13. $\frac{d}{dx}\int_1^x \sin(\sqrt{t})\,dt$ 14. $\frac{d}{dx}\int_{x^2}^{x^3} \sin^2 t\,dt$

15-16. Given $y = x^2 + 2x$, $a=2$, $b=6$, compute the area under y over the interval $[a,b]$ using n rectangles

a) circumscribed b) inscribed.

c) Evaluate each of the above if $n=4$.

d) Find the limit as $n \to \infty$.

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1. $\int x(1+x^3)\,dx$
$= \int x + x^4\,dx$
$= \frac{x^2}{2} + \frac{x^5}{5} + c$

2. $\int \frac{\sin x}{\cos^2 x}\,dx$ let $u = \cos x$, $du = -\sin x\,dx$
$= \int u^{-2}\frac{du}{-1}$
$= \frac{u^{-1}}{-1\cdot -1} + c = \frac{1}{\cos x} + c$
$= \sec x + c$

3. $\int \frac{3x\,dx}{\sqrt{4x^2+5}}$ let $u = 4x^2+5$, $du = 8x\,dx$, $\frac{du}{8} = x\,dx$
$= \frac{3}{8}\int \frac{du}{u^{1/2}}$
$= \frac{3}{8}\int u^{-1/2}du = \frac{3}{8}\frac{2u^{1/2}}{1} + c$
$= \frac{3}{4}(4x^2+5)^{1/2} + c$

4. $\int x^2\sec^2(x^3)\,dx$ let $u = x^3$, $du = 3x^2dx$
$= \int \sec^2 u\,\frac{du}{3}$
$= \frac{1}{3}\tan u + c$
$= \frac{1}{3}\tan x^3 + c$

5. $\int_1^3 \left(x^2 + \frac{1}{x^2}\right)dx$
$= \frac{x^3}{3} + \frac{x^{-1}}{-1}\Big|_1^3$
$= \left(9 - \frac{1}{3}\right) - \left(\frac{1}{3} - 1\right)$
$= 9 - \frac{1}{3} - \frac{1}{3} + 1 = \frac{28}{3}$

6. $\int_4^9 2y\sqrt{y}\,dy = \int_4^9 2y^{3/2}dy$
$= \frac{2\cdot 2y^{5/2}}{5}\Big|_4^9$
$= \frac{4}{5}\left(9^{5/2} - 2^{5/2}\right)$
$= \frac{4}{5}\left(3^5 - 2^5\right) = \frac{4}{5}(243-32)$
$= \frac{4}{5}\cdot 211 = \frac{844}{5}$

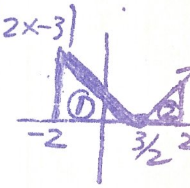
7. $\int_0^8 x\sqrt{1+x}\,dx$ let $u = 1+x$, $du = dx$, $x = u-1$; if $x=0$, $u=1$; $x=8$, $u=9$
$= \int_1^9 (u-1)u^{1/2}du$
$= \int_1^9 \left(u^{3/2} - u^{1/2}\right)du$
$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^9$
$= \frac{2}{5}\cdot 3^5 - \frac{2}{3}\cdot 3^3 - \left(\frac{2}{5} - \frac{2}{3}\right)$
$= \frac{486}{5} - \frac{54}{3} - \frac{2}{5} + \frac{2}{3}$
$= \frac{484}{5} - \frac{52}{3} = \frac{1452-260}{15} = \frac{1192}{15}$

8. $\int_0^{\pi/4} \tan^3 x\sec^2 x\,dx$ let $u = \tan x$, $du = \sec^2 x\,dx$; if $x=0$, $u=0$; $x=\pi/4$, $u=1$
$= \int_0^1 u^3 du$
$= \frac{u^4}{4}\Big|_0^1 = \frac{1}{4}$

9. $1+3+5+ \ldots 23$
$\sum_{i=0}^{11}(2i+1)$ or $\sum_{i=1}^{12}(2i-1)$

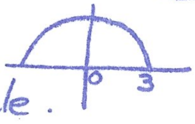
10. $\sum_{k=50}^{100} k^2 = \sum_{k=1}^{100} k^2 - \sum_{k=1}^{49} k^2$
$\left(\sum_{k=1}^{n} = \frac{n(n+1)(2n+1)}{6}\right) = \frac{100(101)(201)}{6} - \frac{49\cdot 50(99)}{6}$
$= 338{,}350 - 40{,}425 = 297{,}925$

11. $y = |2x-3|$



$f(-2) = 7$
$f(2) = 1$
$\Delta\#1\ A = \frac{1}{2}\cdot\frac{7}{2}\cdot 7 = \frac{49}{4}$
$\Delta\#2\ A = \frac{1}{2}\cdot\frac{1}{2}\cdot 1 = \frac{1}{4}$
$A_{Total} = \frac{50}{4} = \frac{25}{2}$

12. $y = \sqrt{9-x^2}$



upper half circle.
$A = \pi r^2$
Area Quarter circle $= \frac{\pi\cdot 3^2}{4}$ or $\frac{9\pi}{4}$

13. $\frac{d}{dx}\int_1^x \sin\sqrt{t}\,dt = \sin\sqrt{x}$

14. $\frac{d}{dx}\int_{x^2}^{x^3}\sin^2 t\,dt = \frac{d}{dx}\int_{x^2}^{a}\sin^2 t\,dt + \frac{d}{dx}\int_a^{x^3}\sin^2 t\,dt$
$x^2 = u$, $x^3 = v$
$= -\frac{d}{dx}\int_a^{x^2}\sin^2 t\,dt + \frac{d}{dx}\int_a^{x^3}\sin^2 t\,dt.$
$= -\frac{d}{du}\left(\int_a^u \sin^2 t\,dt\right)\frac{du}{dx} + \frac{d}{dv}\left(\int_a^v \sin^2 t\,dt\right)\frac{dv}{dt}$
$= -\sin^2 u\cdot 2x + \sin^2 v\cdot 3x^2 = -\sin^2(x^2)\cdot 2x + \sin^2(x^3)\cdot 3x^2$

15. $y = x^2 + 2x$ $a=2$, $b=6$ $\Delta x = \frac{b-a}{n} = \frac{4}{n}$
a) $i = 1$ to n $x_i = a + i\Delta x$
$A = \sum f(x_i)\Delta x$ $x_i = 2 + \frac{4i}{n}$
$= \sum_1^n\left[\left(2+\frac{4i}{n}\right)^2 + 2\left(2+\frac{4i}{n}\right)\right]\Delta x$
$= \sum_1^n\left[4 + \frac{16i}{n} + \frac{16i^2}{n^2} + 4 + \frac{8i}{n}\right]\frac{4}{n}$
$= \sum_{i=1}^n \frac{32}{n} + \sum_{i=1}^n \frac{96i}{n^2} + \sum_{i=1}^n \frac{64i^2}{n^3}$
$= \frac{32}{n}\cdot n + \frac{96}{n^2}\frac{n(n+1)}{2} + \frac{64}{n^3}\frac{n(n+1)(2n+1)}{6}$
$= 32 + 48\frac{n+1}{n} + \frac{64}{n^2}\frac{(n+1)(2n+1)}{6}$

15a) Continued: CALCULUS I UCF

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$$A = 32 + 48\left(\frac{n+1}{n}\right) + \frac{32}{3}\frac{(n+1)(2n+1)}{n^2}$$

b) $\sum_{i=0}^{n-1} f(x_i)\Delta x$

c) $32 + \cancel{48}^{12}\cdot\frac{5}{\cancel{4}} + \frac{\cancel{32}^{2}}{3}\cdot\frac{5(\cancel{9})^{3}}{\cancel{16}}$ $(n=4)$

$= 32 + 60 + 30 = 122$

d) $\lim_{n\to\infty} = 32 + 48 + \frac{32}{3}\cdot 2$

$= 80 + \frac{64}{3} = \frac{240+64}{3} = \frac{304}{3}$

Alt. Problem: Find all values of x^* such that the Mean Value Theorem for Integrals is satisfied: $f(x) = 3x^2$ in $[-3,1]$

$$f(x^*) = \frac{1}{b-a}\int_a^b f(x)\,dx, \text{ where } x^* \text{ in } (a,b)$$

$$= \frac{1}{4}\int_{-3}^{1} 3x^2\,dx$$

$$= \frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}}\Big|_{-3}^{1}$$

$$= \frac{1}{4}(1-(-27))$$

$$= 7$$

However $f(x^*) = 3(x^*)^2 = 7$

$$(x^*)^2 = \frac{7}{3}$$

$$x^* = \pm\sqrt{\frac{7}{3}}$$

$x^* = \sqrt{\frac{7}{3}}$ Not in $(-3,1)$

$x^* = -\sqrt{\frac{7}{3}}$ is in $(-3,1)$