CALCULUS "I" UCF "MINI-TEST" Sections 4.1 – 4.7 ANTON

Dr. RAPALJE

"Gold sheet" and calculator allowed. Non-graphics calculators only.

1. Sand is falling on a conical pile whose height is always equal to the diameter. If height increases at a constant rate of 5 ft/min, how fast ($\frac{dV}{dt}$) is the volume of sand falling on the pile when the pile is 10 feet high?

2. Determine open intervals for which $f(x) = x^{1/3}(x+4)$ is a) increasing b) concave down. c) Find all critical points d) stationary points e) points of inflection.

In 3–5, sketch the graph.

3. $y = \frac{8(x-2)}{x^2}$

$y' = \frac{8(4-x)}{x^3}$

$y'' = \frac{16(x-6)}{x^4}$

4. $y = \frac{(x-1)^2}{x^2}$

$y' = \frac{2(x-1)}{x^3}$

$y'' = \frac{2(3-2x)}{x^4}$

5. $y = x^{2/3}(x-5)$

$y' = \frac{5(x-2)}{3x^{1/3}}$

$y'' = \frac{10(x+1)}{9x^{4/3}}$

For #3-5 above, give x coordinates of all:

a) critical points.
b) stationary points.
c) vertical asymptotes.
d) vertical tangents.
e) Relative maximums
f) Relative minimums
g) Points of ~~inflection~~.
h) Horizontal asymptotes.

6. Find maximum & minimum values of $f(x) = 4x^3 - 3x^4$

a) on $[-1, 2]$ b) on $(-1, 2)$ c) on $(-\infty, \infty)$

7. An open box is to be formed from a 12 in. square by cutting squares of equal size from each corner and folding up the sides. What size square will result in maximum volume of the box?

CALCULUS I MINI TEST UCF

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1. $V = \frac{1}{3}\pi r^2 h$, $h = 2r$

$V = \frac{1}{3}\pi \frac{h^3}{4}$ $\quad \frac{dh}{dt} = 5$ ft/min.

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$= \frac{\pi}{4} \cdot 10^2 \cdot 5$ ft³/min.

$= 125\pi$ ft³/min.

2. $f(x) = x^{1/3}(x+4)$

$= x^{4/3} + 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$

$= \frac{4}{3}x^{-2/3}(x+1)$

$f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3}$

$= \frac{4}{9}x^{-5/3}(x-2)$

f′ – – – 0 + ∞ + + + + + + +
(−1)

f″ + + + + + ∞ – – 0 + + + + +
(0, 2)

a) Increasing $(-1,0) \cup (0,\infty)$

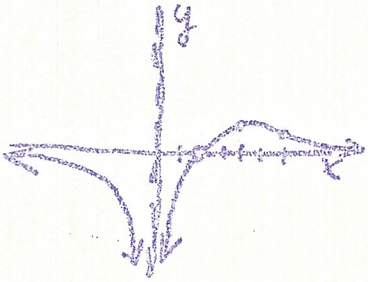
b) Concave down $(0,2)$

c) Critical pts $(0,0)$ and $(-1,-3)$

d) Stat. pts $(-1,-3)$

e) Inflection: $(0,0)$ $(2, 6\sqrt[3]{2})$

3.



x	y
0	–
2	0
4	1
6	8/9

y′ – – – – ∞ + + + 0 – – – – – –
(0, 4)

y″ – – – – ∞ – – – – – – 0 + + + + +
(0, 6)

$\lim_{x \to \pm\infty} f(x) = 0$

a) Critical pts: (4,1)

b) Stationary pts: (4,1)

c) Vert. asymp: x=0

d) Vert. tang: None

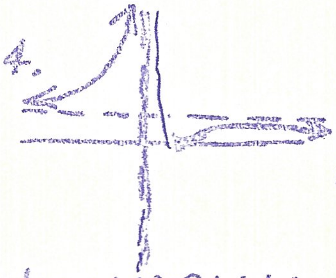
e) Rel max: (4,1)

f) Rel min: None

g) Pt infl: $(6, \frac{8}{9})$

h) Horiz asymp: x axis or y=0

4.



x	y
0	–
1	0
3/2	1/9

y′ + + + + ∞ – 0 + + + +
(0, 1)

y″ + + + + ∞ + + 0 – – – –
(0, 3/2)

$\lim_{x \to \pm\infty} f(x) = 1$

a) Critical: (1,0)

b) Stationary: (1,0)

c) Vert asymp: x=0

d) Vert tang: None

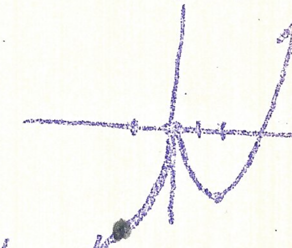
e) Rel max: None

f) Rel min: (1,0)

g) Pt infl: (3/2, 1/9)

h) Horiz. asymp: y=1

5.



x	y
0	0
2	$-3\sqrt[3]{4}$
−1	−6

y′ + + + + ∞ – 0 + + + +
(0, 2)

y″ – – – 0 + ∞ + + + + +
(−1, 0)

$\lim_{x \to -\infty} f(x) = -\infty$ $\quad \lim_{x \to +\infty} f(x) = +\infty$

a) Critical (0,0) $(2, -3\sqrt[3]{4})$

b) Stationary $(2, -3\sqrt[3]{4})$

c) Vert asymp. None.

d) Vert tang: (0,0)

e) Rel max: (0,0)

f) Rel min: $(2, -3\sqrt[3]{4})$

g) Pt infl: (−1, −6)

h) Horiz. asymp: None.

6. $f(x) = 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1-x) = 0$

$x=0$, $x=1$

a) $[-1,2]$ $\quad f(-1) = -7$

$f(2) = -16$ MIN.

$f(0) = 0$

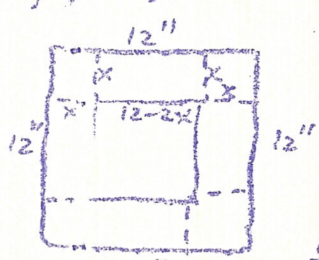
$f(1) = 1$ MAX

b) $(-1,2)$ $f(1) = 1$ MAX

No MIN

c) $(-\infty,\infty)$ $\lim_{x\to\infty} = -\infty$ $\quad \lim_{x\to-\infty} = -\infty$ $\quad$ f(1) = max, No min.

7.



$V = (12-2x)^2(x)$

$= (144 - 48x + 4x^2)x$

$= 144x - 48x^2 + 4x^3$

$\frac{dV}{dx} = 144 - 96x + 12x^2 = 0$

$12(x^2 - 8x + 12) = 0$

$(x-6)(x-2) = 0$

$\cancel{x=6}$ why?

$x = 2''$

$\frac{d^2V}{dx^2} = -96 + 24x$

at $x=2$ $\frac{d^2V}{dx^2} < 0 \Rightarrow$ Rel Max.