CALCULUS I EXAM 2 A R^2 **NAME** _____

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers. Where calculators are used, describe window, procedures, etc.

In 1 - 6, find the derivative f'(x). Be sure answers are simplified.

1. $f(x) = 5\sqrt{x}(x^2 + 4x)$ **[Hint: Is the product rule necessary here?]**

2. $f(x) = \dfrac{3x - 5}{x^2 + 1}$

3. $f(x) = \dfrac{1}{\sqrt{x^2 + 4x}}$

4. $f(x) = \cos^3 5x$

5. $f(x) = \sqrt{x^2 + 1}(2x - 3)^4$ **[Factor completely!]**

6. $f(x) = \sin(x^2) + \sin^2 x$

7. **Use the limit definition of derivative** $\lim_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta X}$ **to show that if f(x) = cos x, then** $f'(x) = -\sin x$.

8. **Use the quotient rule and a trig identity to show that if y = cot x, then** $\dfrac{dy}{dx} = -\csc^2 x$

In 9 - 10, find $\dfrac{dy}{dx}$ **by implicit differentiation.**

9. $x^3 + y^3 = x^3 y^3$

10. y **= sin (**xy**)**

11. Use the calculator to evaluate the derivative of the function $f(x) = \dfrac{3x^2}{x^2 + 2x - 1}$ **a) at** $x = 1$ **b) at** $x = 4$**. Explain your work.**

12. Find the equation of the tangent line to the function $f(x) = x\sqrt{x^2 + 5}$ **at the point (2,6). Explain your work.**

13. If $f(x) = \dfrac{1}{x}$**, find** $f^{(20)}(x)$**. [Do a few derivatives and generalize.]**

14. Sand is falling off a conveyor belt onto a conical pile at the rate of 20 cubic feet per minute. If the diameter of the pile is equal to the height of the pile at any time, at what rate is the height of the pile changing when the pile is 15 feet high?

15. A man 6 feet tall walks at a rate of 10 ft per sec away from a light that is 20 feet above the ground. How fast is his shadow lengthening when he is 15 feet from the base of the light?

CALCULUS I EXAM 2A Solutions

1. $f(x) = 5\sqrt{x}(x^2+4x)$
$= 5x^{1/2}(x^2+4x)$
$= 5x^{5/2} + 20x^{3/2}$
$f'(x) = \frac{25}{2}x^{3/2} + 30x^{1/2}$

2. $f(x) = \frac{3x-5}{x^2+1}$
$f'(x) = \frac{(x^2+1)\cdot 3 - (3x-5)\cdot 2x}{(x^2+1)^2}$
$= \frac{3x^2+3-6x^2+10x}{(x^2+1)^2}$
$= \frac{-3x^2+10x+3}{(x^2+1)^2}$

3. $f(x) = \frac{1}{\sqrt{x^2+4x}} = (x^2+4x)^{-1/2}$
$f'(x) = -\frac{1}{2}(x^2+4x)^{-3/2}(2x+4)$
$= \frac{-(x+2)}{(x^2+4x)^{3/2}}$

4. $f(x) = (\cos 5x)^3$
$f'(x) = 3(\cos 5x)^2(-\sin 5x)(5)$
$= -15\sin 5x \cos^2 5x$

5. $f(x) = \sqrt{x^3+1}\,(2x-3)^4$
$f'(x) = \sqrt{x^3+1}\,4(2x-3)^3\cdot 2 + (2x-3)^4\frac{1}{2}(x^3+1)^{-1/2}\cdot 3x^2$
$= (x^3+1)^{-1/2}(2x-3)^3\left[8(x^3+1) + (2x-3)x\right]$
$= (x^3+1)^{-1/2}(2x-3)^3\left[8x^3+8+2x^2-3x\right]$
$= \frac{(2x-3)^3\left[10x^2-3x+8\right]}{(x^3+1)^{1/2}}$

6. $f(x) = \sin(x^2) + \sin^2 x$
$f'(x) = \cos(x^2)(2x) + 2\sin x\cos x$
$= 2x\cos(x^2) + \sin 2x$

7. $f(x) = \cos x$
$f(x+\Delta x) = \cos(x+\Delta x)$
$f'(x) = \lim_{\Delta x\to 0}\frac{\cos(x+\Delta x)-\cos x}{\Delta x}$
$= \lim_{\Delta x\to 0}\frac{\cos x\cos\Delta x - \sin x\sin\Delta x - \cos x}{\Delta x}$
$= \lim_{\Delta x\to 0}\left[\frac{\cos x\cos\Delta x - \cos x}{\Delta x} - \frac{\sin x\sin\Delta x}{\Delta x}\right]$
$= \lim_{\Delta x\to 0}\left[\frac{\cos x(\cos\Delta x - 1)}{\Delta x} - \sin x\frac{\sin\Delta x}{\Delta x}\right]$
$= \cos x\cdot\lim_{\Delta x\to 0}\left[-\frac{(1-\cos\Delta x)}{\Delta x}\right] - \sin x\lim_{\Delta x\to 0}\left[\frac{\sin\Delta x}{\Delta x}\right]$
$= \cos x\cdot 0 - \sin x\cdot 1 = -\sin x$

8. $y = \cot x = \frac{\cos x}{\sin x}$
$\frac{dy}{dx} = \frac{\sin x\cdot(-\sin x) - \cos x\cos x}{(\sin x)^2}$
$= \frac{-\sin^2 x - \cos^2 x}{(\sin x)^2} = -\frac{1}{\sin^2 x}$
$= -\csc^2 x$

9. $x^3 + y^3 = x^3y^3$
$3x^2 + 3y^2\frac{dy}{dx} = x^3\cdot 3y^2\frac{dy}{dx} + y^3\cdot 3x^2$
$3y^2\frac{dy}{dx} - 3x^3y^2\frac{dy}{dx} = 3x^2y^3 - 3x^2$
$3y^2\frac{dy}{dx}(1-x^3) = 3x^2(y^3-1)$
$\frac{dy}{dx} = \frac{x^2(y^3-1)}{y^2(1-x^3)}$

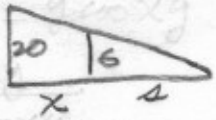
10. $y = \sin(xy)$
$\frac{dy}{dx} = \cos(xy)\left[x\frac{dy}{dx} + y\right]$
$\frac{dy}{dx} = x\frac{dy}{dx}\cos(xy) + y\cos(xy)$
$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$
$\frac{dy}{dx} = \frac{y\cos(xy)}{1-x\cos(xy)}$

11. $f(x) = \frac{3x^2}{x^2+2x-1}$
a) 2nd Calc, der1 ($3x^2/(x^2+2x-1)$, x, 1) = 0
b) 2nd Entry → Replace "1" with "4" = .136 (72/529)

12. (2,6) $f(x) = x\sqrt{x^2+5}$
der1 $(x\sqrt{x^2+5}, x, 2) = 4\frac{1}{3} = \frac{13}{3}$
$y - y_1 = m(x - x_1)$
$y - 6 = \frac{13}{3}(x-2)$
$y - 6 = \frac{13}{3}x - \frac{26}{3} + \frac{18}{3}$
$y = \frac{13}{3}x - \frac{8}{3}$
$13x - 3y = 8$

13. $f(x) = x^{-1}$
$f'(x) = -x^{-2}$
$f''(x) = 2x^{-3}$
$f'''(x) = -3\cdot 2x^{-4}$
$f^{IV}(x) = 4\cdot 3\cdot 2x^{-5}$
$f^{20}(x) = 20!\,x^{-21}$

14. $V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$
$V = \frac{1}{12}\pi h^3$
$\frac{dV}{dt} = 3\cdot\frac{1}{12}\pi h^2\frac{dh}{dt}$
$20 = \frac{1}{4}\pi\cdot 15^2\frac{dh}{dt}$
$d = h = 2r$
$r = \frac{h}{2}$
$\frac{dV}{dt} = 20$
$\frac{dh}{dt} = \frac{80}{225\pi} = \frac{16}{45\pi}$ ft/min

15. 

$\frac{20}{x+6} = \frac{6}{s}$
$20s = 6x + 6s$
$14s = 6x$
$14\frac{ds}{dt} = 6\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{6\cdot 10}{14} = \frac{30}{7}$ ft/sec
$\frac{dx}{dt} = 10$ ft/sec.