MAC 2311 EXAM 2B **R^2** **NAME** ______________________________

Show all work on separate paper. Turn in ALL worksheets.
When you use the calculator, say so, and explain what you did.

1. Find the slope and the equation of the tangent line to the graph $f(x) = x + \frac{1}{x}$. at the point (4,5).

In 2 – 6, find the derivative of each function.

2. $y = \frac{3}{(2x)^3}$.

3. $y = \frac{2}{\sqrt[3]{x}} + 3\cos x$.

4. $y = \left(\frac{3x-5}{x^2+2}\right)^3$ (Express as single fraction, factored completely!)

5. $y = \frac{1}{(x^2+3x-2)^3}$.

6. $y = \sin^3 4x + \tan 2x$

7. Use the limit definition to find the derivative of $f(x) = \frac{4}{\sqrt{x}}$.

8. Use the formula $\tan x = \frac{\sin x}{\cos x}$ and the quotient rule to derive the formula for the derivative of $y = \tan x$.

9. Find the second derivatives of $y = \sin(x^2) + \sin^2 x$.

10. Find the second and third derivatives of $y = 2\sqrt{x}$.
Find $y''(1)$ and $y'''(1)$. Check your answers with graphing calculator.

11. Find $\dfrac{dy}{dx}$ by implicit differentiation: $x^3y^3 - 3y = 6x$.

12. Find $\dfrac{d^2y}{dx^2}$ in terms of x and y by implicit differentiation: $y^2 = x^3$.

13. Find the point(s) at which the graph of the equation $25x^2 + 16y^2 + 200x - 160y + 400 = 0$.

14. Find all values of x on the graph of $f(x) = x^3 - 2x^2 + 5x - 16$ at which the slope of the tangent line is 4.

15. As a balloon in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch? Given: $V = \dfrac{4}{3}\pi r^3$

16. A man 6 feet tall is walks at a rate of 5 ft/sec away from a light that is 15 feet above the ground. When he is 10 feet from the base of the light,

    a) at what rate is the tip of his shadow moving?

    b) at what rate is the shadow lengthening?

MAC2311 EXAM 2B Solutions

1. $f(x) = x + \frac{4}{x}$ or $x + 4x^{-1}$

$f'(x) = 1 - 4x^{-2}$ at $(4,5)$

$f'(4) = 1 - 4 \cdot \frac{1}{16} = \frac{3}{4}$

$y = mx + b$.

$5 = \frac{3}{\cancel{4}}(\cancel{4}) + b$

$2 = b$ $\quad y = \frac{3}{4}x + 2$

2. $y = \frac{3}{(2x)^3} = \frac{3}{8}x^{-3}$

$y' = -\frac{9}{8}x^{-4} = -\frac{9}{8x^4}$

3. $y = \frac{2}{\sqrt[3]{x}} + 3\cos x$

$y = 2x^{-1/3} + 3\cos x$

$y' = -\frac{2}{3}x^{-4/3} - 3\sin x$

$y' = -\frac{2}{3x^{4/3}} - 3\sin x$

4. $y = \left(\frac{3x-5}{x^2+2}\right)^3 = \frac{(3x-5)^3}{(x^2+2)^3}$

$y' = \frac{(x^2+2)^3 3(3x-5)^2 \cdot 3 - (3x-5)^3 3 \cdot (x^2+2)^2 \cdot 2x}{(x^2+2)^6}$

$= \frac{3(x^2+2)^2(3x-5)^2\left[3(x^2+2) - 2x(3x-5)\right]}{(x^2+2)^6}$

$= \frac{3(3x-5)^2\left[3x^2+6-6x^2+10x\right]}{(x^2+2)^4}$

$\frac{3(3x-5)^2(-3x^2+10x+6)}{(x^2+2)^4}$

5. $y = \frac{1}{(x^2+3x-2)^3}$

$y = (x^2+3x-2)^{-3}$

$y' = -3(x^2+3x-2)^{-4}(2x+3)$

$= \frac{-3(2x+3)}{(x^2+3x-2)^4}$

6. $y = \sin^3 4x + \tan 2x$

$y' = 3\sin^2 4x \cdot \cos 4x \cdot 4$
$\quad + \sec^2 2x \cdot 2$

$y' = 12\sin^2 4x \cos 4x + 2\sec^2 2x$

7. $f(x) = \frac{4}{\sqrt{x}}$ $\quad f(x+\Delta x) = \frac{4}{\sqrt{x+\Delta x}}$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \left(\frac{4}{\sqrt{x+\Delta x}} - \frac{4}{\sqrt{x}}\right) \cdot \frac{1}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{(4\sqrt{x} - 4\sqrt{x+\Delta x})}{\sqrt{x+\Delta x}\sqrt{x}\,\Delta x} \frac{(4\sqrt{x} + 4\sqrt{x+\Delta x})}{(4\sqrt{x} + 4\sqrt{x+\Delta x})}$

$= \lim_{\Delta x \to 0} \frac{16x - 16(x+\Delta x)}{\sqrt{x+\Delta x}\sqrt{x}\,\Delta x(4\sqrt{x} + 4\sqrt{x+\Delta x})}$

$= \lim_{\Delta x \to 0} \frac{\cancel{16x} - \cancel{16x} - 16\Delta x}{\sqrt{x+\Delta x}\sqrt{x}\,\Delta x(4\sqrt{x} + 4\sqrt{x+\Delta x})}$

$= \lim_{\Delta x \to 0} \frac{-16\cancel{\Delta x}}{\sqrt{x+\Delta x}\sqrt{x}\,\cancel{\Delta x}(4\sqrt{x} + 4\sqrt{x+\Delta x})}$

$= \frac{-16}{\sqrt{x}\sqrt{x}(4\sqrt{x} + 4\sqrt{x})} = \frac{-16}{x(8\sqrt{x})}$

$= -\frac{2}{x^{3/2}}$ or $-2x^{-3/2}$

8. $y = \tan x = \frac{\sin x}{\cos x}$

$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

9. $y = \sin(x^2) + \sin^2 x$

$y' = \cos(x^2) \cdot 2x + 2\sin x \cos x$

$y'' = \cos x^2 \cdot 2 + 2x \cdot (-\sin(x^2) \cdot 2x$
$\quad + 2\sin x(-\sin x) + \cos x(2\cos x)$

$= 2\cos x^2 - 4x^2 \sin(x^2)$
$\quad - 2\sin^2 x + 2\cos^2 x$

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10. $y = 2x^{1/2}$

$y' = x^{-1/2}$

$y'' = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$

$y''' = \frac{3}{4}x^{-5/2} = \frac{3}{4x^{5/2}}$

$y''(1) = -\frac{1}{2}$ $\quad y'''(1) = \frac{3}{4}$

11. $x^3y^3 - 3y = 6x$

$x^3 3y^2 y' + y^3 \cdot 3x^2 - 3y' = 6$

$y'(3x^3y^2 - 3) = 6 - 3x^2y^3$

$y' = \frac{\cancel{3}(2 - x^2y^3)}{\cancel{3}(x^3y^2 - 1)} = \frac{2 - x^2y^3}{x^3y^2 - 1}$

12. $y^2 = x^3$

$2yy' = 3x^2$

$y' = \frac{3x^2}{2y}$

$y'' = \frac{2y \cdot 6x - 3x^2 \cdot 2y'}{(2y)^2}$

$= \frac{12xy - \cancel{6}^{3}x^2\left(\frac{3x^2}{\cancel{2}y}\right)}{4y^2}$

$= \frac{y\left(12xy - \frac{9x^4}{y}\right)}{y \cdot (4y^2)}$

$= \frac{12xy^2 - 9x^4}{4y^3}$

$\frac{3x}{y} - \frac{9x^4}{4y^3}$

13. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32yy' + 200 - 160y' = 0$

$y'(32y - 160) = -50x - 200$

$y' = \frac{-50(x+4)}{32(y-5)}$

Horiz Tangents $\quad$ Vertical Tangents

$x = -4$ $\quad$ $y = 5$

$\cancel{400} + 16y^2 - 800 - 160y + \cancel{400} = 0$

$16y^2 - 160y = 0$

$16y(y - 10) = 0$

$y = 0 \quad y = 10$

$(-4, 0) \quad (-4, 10)$

$25x^2 + \cancel{400} + 200x - \cancel{800} + \cancel{400} = 0$

$25x(x + 8) = 0$

$x = 0 \quad x = -8$

$(0, 5) \quad (-8, 5)$

14. $f'(x) = x^3 - 2x^2 + 5x - 16$

$f'(x) = 3x^2 - 4x + 5 = 4$

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = 1/3 \quad x = 1$

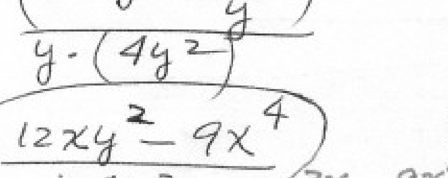
15. $V = \frac{4}{3}\pi r^3$ where $\frac{dV}{dt} = 4\,in^3/sec.$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ at $r = 1\,in.$

$4 = 4\pi 1^3 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{4}{4\pi} = \frac{1}{\pi}$ in/sec.

16. Let y = dist to tip of shadow.



$\frac{dx}{dt} = 5$ ft/sec.

$\frac{s}{6} = \frac{y}{15}$

$\frac{6}{s} = \frac{15}{x+s}$. $\quad \frac{ds}{dt}$ = rate shadow is lengthening.

$6x + 6s = 15s$

b) $\frac{ds}{dt} = \frac{6}{9}\frac{dx}{dt} = \frac{2}{3} \cdot 5 = \frac{10}{3}$ ft/sec

$6x = 9s$

$6\frac{dx}{dt} = 9\frac{ds}{dt}$.

a) $15s = 6y$

$15\frac{ds}{dt} = 6\frac{dy}{dt}$, so $\frac{dy}{dt} = \frac{15 \cdot \frac{10}{3}}{6} = 25/3$ ft/sec