CALCULUS I EXAM 3 A R^2 NAME ________________________

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers.
Where calculators are used, describe window, procedures, etc.

1. **Find all critical numbers for** $f(x) = 3x^4 - 12x^2$
   Find the maximum and minimum values (if they exist) of f(x) on [-1,2).

2. **Given** $f(x) = 10 - \dfrac{16}{x}$ **in [2,8], find all values of c that satisfy the Mean Value Theorem,** $f\,'(c) = \dfrac{f(b) - f(a)}{b - a}$

In 3-4,find all critical numbers, intervals increasing/decreasing, relative maximum/minimum points, points of inflection (if any), intervals concave up/down, vertical asymptotes, vertical tangents, sketch the graph.

3. **Use the graphing calculator** $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$

4. **Use first and second derivative tests** $f(x) = 3x^3 - 2x^{\frac{2}{}}$ **.**

5. **Given the table:**

x		-2		-1		0		1	
f		∞		1		0		1	
f'	-	∞	-	-	-	inf	+	0	-
f''	-	∞	+	0	-	∞	-	-	-

$\lim\limits_{x \to -\infty} f(x) = 0$ **and** $\lim\limits_{x \to \infty} f(x) = -\infty$

Sketch the graph.
Identify critical points,
relative max and mins,
points of inflection,
asymptotes,
and vertical tangents.

6. **Find each of the following limits:**

   a) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 6x^2 + 5}{3 + 5x^3}$ b) $\lim\limits_{x \to -\infty} \dfrac{2x}{\sqrt{(x^2 + 1)}}$

7. **Use algebraic methods to find the exact value of the limit:**
   $\lim\limits_{x \to \infty}(2x - \sqrt{4x^2 + 3x})$ .8

8. **Use Newton's Method to find the root of** $f(x) = -x^3 + 3x^2 - x + 1$.9
   Draw a sketch, give the x values. Is x = 1 a good initial value? Why or why not?

9. **Given** 10 $f(x) = x^2 - 2x - 3$ **, find df and Δf when** x **=2 and** Δx **= 0.1.**

10. **The sum of two numbers is 60. Find the numbers such that the product of the first times the cube of the second is a maximum.**

11. **A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pens. What dimensions should be used for each pen so that the enclosed area will be a maximum?**

CALCULUS I EXAM 3A

1. $f(x) = 3x^4 - 12x^2$

$f'(x) = 12x^3 - 24x$

$= 12x(x^2 - 2) = 0$

Critical numbers: $x = 0, \pm\sqrt{2}$

Interval $[-1, 2)$

$f(-1) = 3 - 12 = -9$

$f(2) = 3 \cdot 16 - 12 \cdot 4 = 0$

$f(0) = 0$ Max

$f(\sqrt{2}) = 3 \cdot 2 - 12 \cdot 2 = -18$ Min.

2. $f(x) = 10 - 16x^{-1}$, $[2, 8]$

$f'(x) = 16x^{-2}$

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$16x^{-2} = \dfrac{8-2}{8-2} = 1$

$\dfrac{16}{x^2} = 1$

$x^2 = 16$

$x = \pm 4$

Only $x = 4$ is in $[2, 8]$

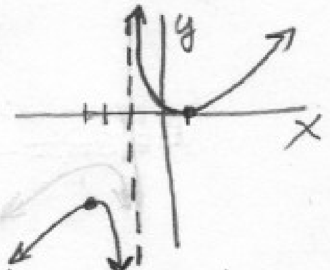
$f(b) = f(8) = 10 - 16 \cdot \frac{1}{8}$

$= 8$

$f(a) = f(2) = 10 - 16 \cdot \frac{1}{2}$

$= 2$

3.



Asymp: $x = -1$

Critical Nos: $x = 1, -3$

Rel Max $(-3, -8)$

Rel Min $(1, 0)$

Concave Up: $(-1, \infty)$

Concave Down: $(-\infty, -1)$

Incr: $(-\infty, -3) \cup (1, \infty)$

Decr: $(-3, -1) \cup (-1, 1)$

4. $f(x) = 3x^{2/3} - 2x$

$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - 2$

$= 2x^{-1/3} - 2$

$= \dfrac{2}{\sqrt[3]{x}} - 2$

$= \dfrac{2 - 2\sqrt[3]{x}}{\sqrt[3]{x}}$

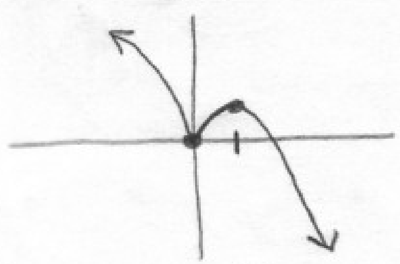
$f'(x) = 0$ at $x = 1$

$f'(x) = \infty$ at $x = 0$

$f(0) = 0$ Vertical Tangent

Critical Nos: $x = 0, 1$

$f''(x) = -\frac{2}{3} x^{-4/3}$



f': $- \ \infty + 0 \ -$

f'': $- \ \infty \ - \ -$

Incr: $(0, 1)$

Decr: $(-\infty, 0) \cup (1, \infty)$

Concave Down $(-\infty, 0) \cup (0, \infty)$

No points of inflection.

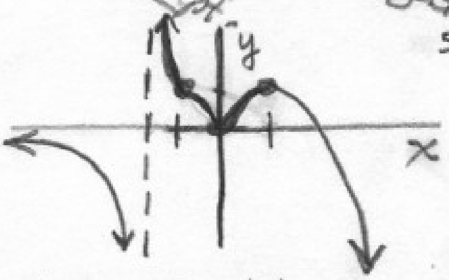
5.

$\lim_{x \to -\infty} = 0$

$\lim_{x \to \infty} = -\infty$

x		-2		-1		0		1	
f		∞		1		0		1	
f'	−	∞	−	−	−	∞	+	0	−
f''	−	∞	+	0	−	∞	−	−	−

Vert Asympt, Inflection, Vert Tangent, Rel Max



5. Critical pts: $x = 0, 1$

Rel Max: $(1, 1)$

Rel min: $(0, 0)$

Pt inflection: $(-1, 1)$

Asymptote: $x = -2$

Vertical tan at $(0, 0)$

6a) $\lim_{x \to \infty} \dfrac{(2x^3 - 6x^2 + 5)\frac{1}{x^3}}{(3 + 5x^3)\frac{1}{x^3}}$

$= \lim_{x \to \infty} \dfrac{2 - 6/x + 5/x^3}{3/x^3 + 5} = \dfrac{2}{5}$

b) $\lim_{x \to -\infty} \dfrac{2x}{\sqrt{x^2 + 1}}$

$= \lim_{x \to -\infty} \dfrac{2x}{\sqrt{x^2(1 + 1/x^2)}}$

$= \lim_{x \to -\infty} \dfrac{x}{|x|} \cdot \dfrac{2}{\sqrt{1 + 1/x^2}}$

$= -1 \cdot 2 = -2$

7. $\lim_{x \to +\infty} (2x - \sqrt{4x^2 + 3x})$

$= \infty - \infty$

$\dfrac{(2x - \sqrt{4x^2 + 3x})}{1} \dfrac{(2x + \sqrt{4x^2 + 3x})}{2x + \sqrt{4x^2 + 3x}}$

$= \dfrac{4x^2 - (4x^2 + 3x)}{2x + \sqrt{4x^2 + 3x}}$

$= \dfrac{-3x}{2x + \sqrt{x^2(4 + 3/x)}}$

$\lim_{x \to \infty} \dfrac{-3x}{2x + x\sqrt{4 + 3/x}}$

$= \lim_{x \to \infty} \dfrac{-3}{2 + \sqrt{4 + 3/x}}$

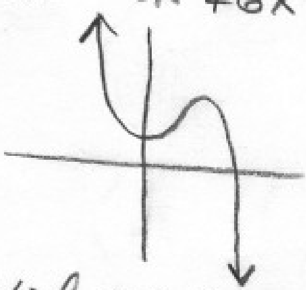
$= \dfrac{-3}{2 + 2} = -\dfrac{3}{4}$

EX3A

$\boxed{PROG}$ $\boxed{NWT2}$ $\boxed{F1}$ (Enter Function)

8. $f(x) = -x^3 + 3x^2 - x + 1$

$f'(x) = -3x^2 + 6x - 1$



Initial $x = 0, 1$ do not work.

$x = 2$
$x = 5$
$x = 3.82608695652$
$x = 3.14671901374$
$x = 2.84232627714$
$x = 2.77284763644$
$x = 2.769301397744$
$x = 2.7692923543$
$x = 2.769292354 24$

$\boxed{PROG}$ $\boxed{DIFF}$ $\boxed{F1}$ (Enter function)

9. $f(x) = x^2 - 2x - 3$

$f'(x) = 2x - 2$

$\boxed{Diff}$

$x = 2$

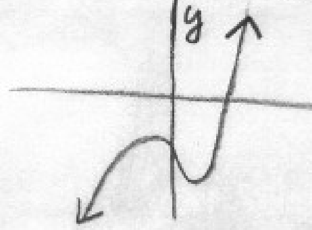
$\Delta x = 0.1 = dx$

$df = 0.2$

$\Delta f = 0.21$

$f(2.1) = (2.1)^2 - 2(2.1) -$
$= -2.79$

$f(2) = -3$



$df = f'(x) \cdot dx$
$= 2\ (0.1) = 0.2$

$\Delta f = f(x + \Delta x) - f(x)$
$= -2.79 - (-3) = 0.21$

10. Let $x = 1^{st}$ no; $y = 2^{nd}$ no.

Primary equation: $P = x \cdot y^3$

Secondary eq: $x + y = 60$

$y = 60 - x$

$P(x) = x(60-x)^3$

$P'(x) = x \cdot 3(60-x)^2(-1) + (60-x)^3 \cdot 1$

$= (60-x)^2(-3x + 60 - x) = 0$

$x = 60$ $\quad$ $-4x + 60 = 0$

$y = 0$ $\quad$ $4x = 60$

Rel min.

$x = 15$
$y = 45$
Rel max

—OR—

$P(x,y) = x \cdot y^3$ $\quad$ $x = 60 - y$.

$P(y) = (60-y)y^3$

$P(y) = 60y^3 - y^4$

$P'(y) = 180y^2 - 4y^3$

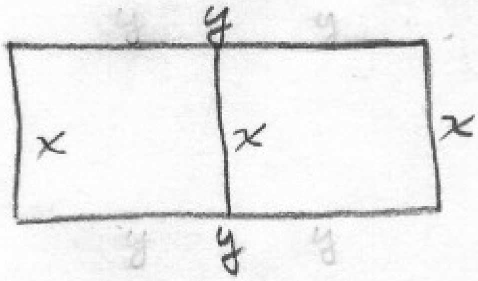
$4y^2(45 - y) = 0$

$y = 0$ $\quad$ $y = 45$

$x = 60$ $\quad$ $x = 15$

min. $\quad$ Max

11.



Primary Equation $A = xy$

Secondary Eq: $3x + 2y = 160$

$2y = 160 - 3x$

$y = \frac{160 - 3x}{2}$

$A = x\left(\frac{160 - 3x}{2}\right)$

$= 80x - \frac{3}{2}x^2$

$A' = 80 - 3x = 0$

$80 = 3x$

$x = 80/3$ ft.

$y = \frac{160 - 80}{2} = 40$ ft

$\boxed{\frac{80}{3}' \text{ by } 40'}$