MAC 2311 EXAM 4B R^2 NAME ______________________

Show all work on separate paper. Turn in ALL worksheets. Give all irrational answers in exact (radical) form. When you use the calculator, say so, and explain what you did.

In 1 – 4, evaluate the indefinite integrals. Show all work!

1. $\int \frac{x^5+2x^2-1}{x^4}\,dx$

2. $\int x^2\sqrt{x^3+1}\,dx$

3. $\int \frac{x-4}{(x^2-8x+1)^2}\,dx$

4. $\int \cos^3 5x \ \sin 5x\,dx$

In 5 – 7, evaluate the definite integrals. Show all work by algebraic techniques!
You may check answers with calculator methods.

5. $\int_1^4 \frac{x-2}{\sqrt{x}}\,dx$

6. $\int_4^{12} x\sqrt{x-3}\,dx$

7. $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$

8. Find f(x) such that $f''(x)=x^{-3/2}, f'(4)=2, f(4)=0$.

9. Sketch the region whose area is indicated by the integral $\int_0^5 \sqrt{25-x^2}\,dx$.
Then use a geometric formula to evaluate the integral.

10. If $\int_2^6 f(x)\,dx=10$ and $\int_2^6 g(x)\,dx=-2$, then find $\int_2^6 [2f(x)+3g(x)]\,dx$.

11. Find the average value of $f(x)=4-x^2$ on [0, 2]. Find all values of x that satisfy the Mean Value Theorem for Integrals
[that is, $\int_a^b f(x)\,dx = f(c)\,(b-a)$].

12. Find the value of $\lim_{x\to\infty}\sum_{i=1}^{n}\left(1-\frac{2i}{n}\right)^2\frac{3}{n}$. Use the formulas and show work by algebra methods.

$\sum_{i=1}^{n} c = cn$ $\qquad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2x+1)}{6}$

13. Hand In Problem (solved and printed from a website on the internet). Use the Trapezoidal Rule and Simpson's Rule to find the approximate value of $\int_0^2 x\sqrt{x^2+1}\,dx$ for n=4 and also for n=10.

14. Find the actual value of $\int_0^2 x\sqrt{x^2+1}\,dx$ (previous exercise!) by integration using algebraic methods, and check by using the calculator integration.

MAC 2311 EXAM 4B Solutions

1. $\int \frac{x^5+2x^2-1}{x^4}\,dx$

$= \int (x + 2x^{-2} - x^{-4})\,dx$

$= \frac{x^2}{2} + \frac{2x^{-1}}{-1} - \frac{x^{-3}}{-3} + c$

$= \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + c$

2. $\int x^2\sqrt{x^3+1}\,dx$ let $u = x^3+1$

$du = 3x^2dx$

$\frac{du}{3} = x^2dx$

$= \int u^{1/2}\frac{du}{3}$

$= \frac{1}{3}\frac{2u^{3/2}}{3} + c$

$= \frac{2}{9}(x^3+1)^{3/2} + c$

3. $\int \frac{x-4}{(x^2-8x+1)^2}\,dx$ let $u = x^2-8x+1$

$du = (2x-8)dx$

$du = 2(x-4)dx$

$\frac{du}{2} = (x-4)dx$

$= \int \frac{\frac{du}{2}}{u^2}$

$= \frac{1}{2}\int u^{-2}du$

$= \frac{1}{2}\,\frac{u^{-1}}{-1} + c$

$= \frac{-1}{2(x^2-8x+1)} + c$

4. $\int \cos^3 5x \sin 5x\,dx$ let $u = \cos 5x$

$du = -(\sin 5x)5dx$

$du = -5\sin 5x\,dx$

$\frac{du}{-5} = \sin 5x\,dx$

$= \int u^3\frac{du}{-5}$

$= -\frac{1}{5}\frac{u^4}{4} + c$

$= \frac{-1}{20}\cos^4 5x + c$

5. $\int_1^4 \frac{x-2}{\sqrt{x}}\,dx$

$\int_1^4 (x^{1/2} - 2x^{-1/2})\,dx$

$\frac{2x^{3/2}}{3} - 2\,\frac{2}{1}x^{1/2}\Big|_1^4$

$\left(\frac{2}{3}\cdot 8 - 4\cdot 2\right) - \left(\frac{2}{3}\cdot 1 - 4\cdot 1\right)$

$\frac{16}{3} - 8 - \frac{2}{3} + 4$

Check: $\frac{14}{3} - \frac{12}{3} = \frac{2}{3}$

fnInt$((x-2)\div\sqrt{x}, x, 1, 4) = .6\overline{6}6$

6. $\int_4^{12} x\sqrt{x-3}\,dx$ let $u = x-3$

$du = dx$

$x = u+3$

$\int (u+3)\,u^{1/2}du$

$\int (u^{3/2} + 3u^{1/2})\,du$

$\frac{2}{5}u^{5/2} + 3\cdot\frac{2}{3}u^{3/2}$

$\frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2}\Big|_4^{12}$

$\left(\frac{2}{5}\cdot 9^{5/2} + 2\cdot 9^{3/2}\right) - \left(\frac{2}{5}\cdot 1^{5/2} + 2\cdot 1^{3/2}\right)$

$\frac{2}{5}\cdot 243 + 2\cdot 27 - \frac{2}{5} - 2$

$\frac{484}{5} + 52 = \frac{744}{5}$ or 148.8

Check: fnInt$(x\sqrt{(x-3)}, x, 4, 12) = 148.8$

7. $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$ let $u = 2x+1$

$du = 2dx$

$\frac{du}{2} = dx$

$= \int \frac{1}{u^{1/2}}\frac{du}{2}$

$= \frac{1}{2}\int u^{-1/2}du$

$= \frac{1}{2}\frac{2}{1}u^{1/2}$

$= (2x+1)^{1/2}\Big]_0^4 = \sqrt{9} - 1 = 2$

fnInt

8. $f''(x) = x^{-3/2}$ $f'(4) = 2$ $f(4) = 0$

$f'(x) = -\frac{2}{1}x^{-1/2} + C_1$

$2 = -2\cdot 4^{-1/2} + C_1$

$2 = -2\cdot\frac{1}{2} + C_1$

$2 = -1 + C_1$ $C_1 = 3$

$f'(x) = -2x^{-1/2} + 3$

$f(x) = -2\cdot\frac{2}{1}x^{1/2} + 3x + C_2$

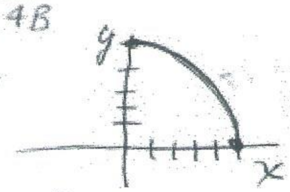
$0 = -4\cdot 4^{1/2} + 3\cdot 4 + C_2$

$0 = -8 + 12 + C_2$ $C_2 = -4$

$f(x) = -4\sqrt{x} + 3x - 4$

EX 4B

9.



$$\int_0^5 \sqrt{25-x^2}\,dx$$

$= \text{Quarter circle}$

$= \frac{1}{4}\pi r^2 = \frac{25\pi}{4} \approx 19.634$

10. $\int_2^6 f(x)\,dx = 10 \quad \int_2^6 g(x)\,dx = -2$

$$\int_2^6 [2f(x) + 3g(x)]\,dx$$

$$= 2\int_2^6 f(x)\,dx + 3\int_2^6 g(x)\,dx$$

$$= 2 \cdot 10 + 3(-2)$$

$$= 20 - 6 = 14$$

11. $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$f_{\text{average}} = \frac{1}{2-0}\int_0^2 (4-x^2)\,dx$$

$$= \frac{1}{2}\left[4x - \frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{2}\left[8 - \frac{8}{3}\right] = \frac{1}{2}\cdot\frac{16}{3}$$

$$= \frac{8}{3}$$

$$f(c) = 4 - c^2 = \frac{8}{3}$$

$$4 - \frac{8}{3} = c^2$$

$$\frac{4}{3} = c^2$$

$$c = \pm\frac{2}{\sqrt{3}}$$

c must be in $[0,2]$ $\quad c = \frac{2}{\sqrt{3}}$

12. $$\sum_{i=1}^{n}\left(1-\frac{2i}{n}\right)^2 \frac{3}{n} = \sum_{i=1}^{n}\left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right)\frac{3}{n}$$

$$= \sum_{i=1}^{n}\frac{3}{n} - \sum_{i=1}^{n}\frac{12i}{n^2} + \sum_{i=1}^{n}\frac{12i^2}{n^3}$$

$$= \frac{3}{n}\sum_{i=1}^{n} 1 - \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{12}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{3}{n}\cdot n - \frac{12}{n^2}\frac{n(n+1)}{2} + \frac{12}{n^3}\frac{n(n+1)(2n+1)}{6}$$

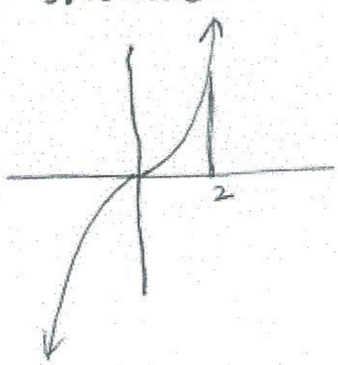
$$= 3 - 6\,\frac{n+1}{n} + 2\,\frac{(n+1)(2n+1)}{n^2}$$

$$\lim_{n\to\infty} = 3 - 6 + 2\cdot 2$$

$$= 1$$

13.

	TRAPEZOID	SIMPSON
$n=4$	3.456730995	3.392214
$n=10$	3.403536	3.3934197



14. $A = \int_0^2 x\sqrt{x^2+1}\,dx$ Let $u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$= \int u^{1/2}\frac{du}{2}$$

$$= \frac{1}{2}\frac{2}{3}u^{3/2}$$

$$= \frac{1}{3}(x^2+1)^{3/2}\Big|_0^2$$

$$= \frac{1}{3}\left[5^{3/2} - 1\right] = \frac{5\sqrt{5}-1}{3}$$

Actual Area

$\approx 3.393446629 17$

Calculator: $f_n\text{Int}(x\sqrt{(x^2+1)}, x, 0, 2)$

$\approx 3.393446629 17$