CALCULUS I EXAM 1E

PRACTICE TESTS AND SOLUTIONS

Dr. Rapaljе)

from Larson + Hostetler, 3rd ed.

Show all work on separate paper. (5 ea)

1. Solve for x and graph on numberline.

$$x > \frac{4}{x}$$

2. Find x so that the distance from $(2,-1)$ to $(x,2)$ is 5.

3. Test for symmetry (x axis, y axis, origin) and give
(8) x and y intercepts: $y = x^3 - 4x$

4. Find the equation of the line joining the points of intersection of
$y = x^2 - 4x + 3$
$y = -x^2 + 2x + 3$

5. Given $f(x) = x^2 - 3x + 7$, find $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

6. Given $f(x) = \dfrac{1}{2x+3}$ and $g(x) = x^2 - 4x$, find $f[g(x)]$ and $g[f(x)]$.
(3 ea)

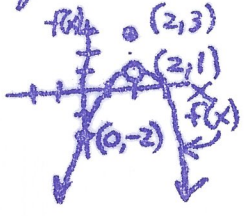
7. Given $f(x) = \sqrt{x^2 - 3x - 4}$, find Domain and range.
(3 ea)

8. Graph $y = \sqrt{x}$. From this, sketch a) $y = \sqrt{x} + 2$ b) $y = -\sqrt{x}$
(2 ea) c) $y = \sqrt{-x}$ d) $y = \sqrt{x+4}$

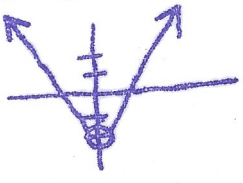
9. Given $\sin\theta = \frac{2}{3}$, draw a triangle and find the exact value of $\tan\theta$ (by finding the missing side)

In 10–16, find the given limit or state DNE (Does not exist.)

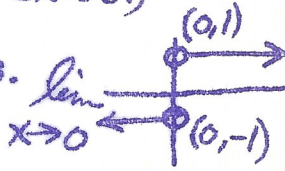
10. $\lim_{x \to 2} f(x)$
(3 ea)



11. $\lim_{x \to 0}$



13. $\lim_{x \to 0}$



14. $\lim_{x \to -2} \dfrac{x^2 - 4}{x^3 + 8}$

15. $\lim_{x \to 0} \dfrac{\sqrt{4+x} - 2}{x}$

16. $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\theta}$
(4 ea)

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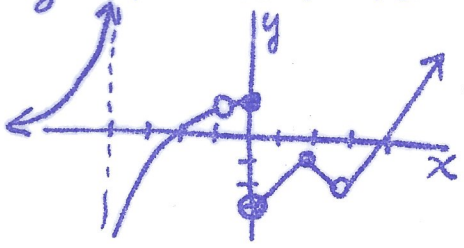
17. For what value(s) of c will

$$f(x) = \begin{cases} cx-1 & x<4 \\ cx^2 & x \geq 4 \end{cases}$$ be continuous?

18. Give all values for which the function is discontinuous:
(Distinguish whether removable or non removable discontinuities.)



19. Use the function $f(x) = \dfrac{x+2}{x^2-2x-8}$ to explain the difference between removable and non-removable discontinuities. Sketch a graph.

20. If $f(x) = \frac{1}{2}x + 4$, $c = 2$, and $\epsilon = .02$, find L, and find δ such that $|f(x) - L| < \epsilon$ for every x where $0 < |x - c| < \delta$.

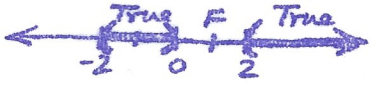
CALCULUS I EXAM 1E Solutions — Dr. RAPALJE

1. $x > \frac{4}{x}$ $x=0$ endpoint.

$x = \frac{4}{x}$

$x^2 = 4$

$x = \pm 2$



2. $\sqrt{(x-2)^2+3^2} = 5$

$(x-2)^2 + 9 = 25$

$(x-2)^2 = 16$

$x-2 = \pm 4$

$x = 2 \pm 4$ → **6a-2**

3. $y = x^3 - 4x$

x axis: $-y = x^3 - 4x$ No.

y axis: $y = (-x)^3 - 4(-x)$

$= -x^3 + 4x$ No.

Origin: $(-y) = (-x)^3 - 4(-x)$

$-y = -x^3 + 4x$

$y = x^3 - 4x$ yes.

y int: $x = 0$

y = 0

x int: $y = 0$

$0 = x(x^2-4)$

x = 0, 2, -2

4. $y = x^2 - 4x + 3$

$y = -x^2 + 2x + 3$

$x^2 - 4x + 3 = -x^2 + 2x + 3$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0 \quad x = 3$

$y = 3 \quad y = 9 - 12 + 3 = 0$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{3-0} = -1$

$y - 3 = -1(x-0)$

$y = -x + 3$ ∼ **$x + y - 3 = 0$**

5. $f(x) = x^2 - 3x + 7$

$f(x+\Delta x) = (x+\Delta x)^2 - 3(x+\Delta x) + 7$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 3x - 3\Delta x + 7 - x^2 + 3x - 7}{\Delta x}$$

$$= \frac{\Delta x(2x + \Delta x - 3)}{\Delta x}$$

$$= \mathbf{2x + \Delta x - 3}$$

6. $f(x) = \frac{1}{2x+3}$ $g(x) = x^2 - 4x$

$f[g(x)] = \frac{1}{2(x^2-4x)+3}$

$= \frac{1}{2x^2 - 8x + 3}$

$g[f(x)] = \left(\frac{1}{2x+3}\right)^2 - 4\left(\frac{1}{2x+3}\right)$

$= \frac{1}{(2x+3)^2} - \frac{4}{(2x+3)}$

7. $f(x) = \sqrt{x^2 - 3x - 4}$

D: $x^2 - 3x - 4 \geq 0$

$(x-4)(x+1) \geq 0$

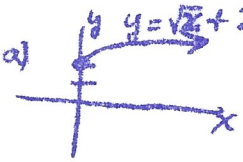
$x = 4 \quad x = -1$

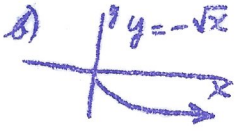
"EXTREMES"

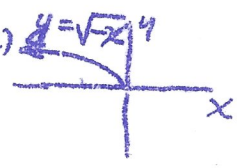
$x \leq -1 \cup x \geq 4$

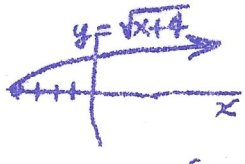
R: $y \geq 0$

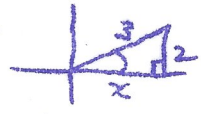
8.

a) 

b) 

c) 

d) 

9. 

$x^2 + 4 = 9$

$x = \sqrt{5}$

$\tan\theta = \frac{2}{\sqrt{5}}$

10. 1

11. -2

12. Missing!!

13. DNE

14. $\lim_{x \to -2} \frac{(x-2)(x+2)}{(x+2)(x^2-2x+4)}$

$= \frac{-4}{4+4+4} = -\frac{1}{3}$

15. $\lim_{x \to 0} \frac{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}{x \quad (\sqrt{4+x} + 2)}$

$= \lim_{x \to 0} \frac{x + x - 4}{x(\sqrt{4+x} + 2)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{4}$

16. $\lim_{\theta \to 0} 3\frac{\sin 3\theta}{3\theta} = 3$

17. $f(x) = \begin{cases} cx - 1 & x < 4 \\ cx^2 & x \geq 4 \end{cases}$

$\lim_{x \to 4^-} f(x) = 4c - 1$

$\lim_{x \to 4^+} f(x) = 16c$

$4c - 1 = 16c$

$-1 = 12c$

$c = -1/12$

18. Removable discontinuities at **$x = -1$ and $x = 3$**

Non removable at **$x = -4, 0$**

19. $f(x) = \frac{x+2}{(x-4)(x+2)}$

$= \frac{1}{x-4}, \quad x \neq -2.$

Removable disc. at $x = -2$ (Hole in graph)

Non removable at $x = 4$ (Asymptote)

$\lim_{x \to -2} f(x) = -\frac{1}{6}$

20. $f(x) = \frac{1}{2}x + 4$

$\lim_{x \to 2} (\frac{1}{2}x + 4) = 5 = L$

$|\frac{1}{2}x + 4 - 5| < .02$

$|\frac{x-2}{2}| < .02$

$|x-2| < .04$

$0 < |x-2| < \delta$

Let **$\delta = .04$**