CALCULUS I EXAM 1D Dr. RAPALJE

Show all work neatly and well organized on separate paper.
Circle answers. (Simplify completely!) (No Calculators yet!)

PART I: (6 points each)

1. Solve for x, graph on numberline: $x^2 - x \geq 12$.
2. Solve for n: $I = \dfrac{nE}{R - nr}$
3. Solve for x: $(2x+1)^2 - 3x^2 = 2x + 7$
4. Solve for x: $|2x-6| = |4-5x|$
5. Solve for y in terms of x: $2xy = y^2 - 4x$
6. If $f(x) = x^2 - 5x + 6$, find $\dfrac{f(x+h) - f(x)}{h}$.
7. If $f(x) = x^2 - 4$ and $g(x) = \dfrac{1}{x+2}$, find $f[g(x)]$ and $g[f(x)]$
8. Factor <u>completely</u>: $12x^2(x^2+4)^5(2x^3-1)^2 - 27(2x^3-1)^3(x^2+4)^4$
9. Simplify by factoring: $x^2(1+x^2)^{-1/2} - \sqrt{1+x^2}$
10. Find the equation of the perpendicular bisector of the line segment from $(2,-4)$ to $(4,-2)$. (Form $Ax+By+C=0$)

PART II:

1. (10 pts) Triangle ABC is formed by points $A(2,-4)$ $B(4,-2)$ $C(8,-10)$. Find a) the <u>length</u> and b) the <u>equation</u> of the <u>median</u> from A to BC. (Give length in simplest radical form and equation $Ax+By+C=0$.)

2. (12 pts) Solve $\left|\dfrac{3-2x}{x+4}\right| < 1$. Use any method you wish, but be complete. Give answer in interval notation, graph on number line.

3. (18 pts) Given $x^2y = x-3$, find a) all intercepts, b) all asymptotes c) domain d) range. e) Symmetry.

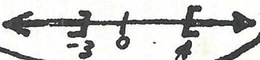
CALCULUS I EXAM 1D Solutions Dr. RAPALJE

1. $x^2 - x - 12 \geq 0$

$(x-4)(x+3) \geq 0$

Extremes with endpts: $x = 4$, $x = -3$

$x \leq -3$ or $x \geq 4$



2. $I = \frac{nE}{R - nr}$

$IR - Inr = nE$

$IR = nE + Inr$

$IR = n(E + Ir)$

$n = \frac{IR}{E + Ir}$

3. $(2x+1)^2 - 3x^2 = 2x + 7$

$4x^2 + 4x + 1 - 3x^2 = 2x + 7$

$x^2 + 2x + 1 = 7$

$(x+1)^2 = 7$

$x + 1 = \pm\sqrt{7}$

$x = -1 \pm \sqrt{7}$

4. $|2x - 6| = |4 - 5x|$

$2x - 6 = 4 - 5x$ or $2x - 6 = -4 + 5x$

$7x = 10$ $\quad$ $-2 = 3x$

$x = 10/7$ or $x = -2/3$

5. $2xy = y^2 - 4x$

$y^2 - 2xy - 4x = 0$

$y = \frac{2x \pm \sqrt{4x^2 + 16x}}{2}$

$= \frac{2x \pm 2\sqrt{x^2 + 4x}}{2}$

$= x \pm \sqrt{x^2 + 4x}$

6. $f(x) = x^2 - 5x + 6$

$f(x+h) = (x+h)^2 - 5(x+h) + 6$

$f(x+h) - f(x) = \cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h + \cancel{6} - \cancel{x^2} + \cancel{5x} - \cancel{6}$

$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 5h}{h}$

$= 2x + h - 5$

7. $f(x) = x^2 - 4$ $\quad$ $g(x) = \frac{1}{x+2}$

$f(g(x)) = \left(\frac{1}{x+2}\right)^2 - 4$ $\quad$ $g(f(x)) = \frac{1}{(x^2-4)+2}$

$= \frac{1}{x^2 - 2}$

8. $12x^2(x^2+4)^5(2x^3-1)^2 - 27(2x^3-1)^3(x^2+4)^4$

$= 3(x^2+4)^4(2x^3-1)^2\left[4x^2(x^2+4) - 9(2x^3-1)\right]$

$3(x^2+4)^4(2x^3-1)^2(4x^4 - 18x^3 + 16x^2 + 9)$

9. $x^2(1+x^2)^{-1/2} - \sqrt{1+x^2}$

$= (1+x^2)^{-1/2}\left[x^2 - (1+x^2)^1\right]$

$= (1+x^2)^{-1/2}\left[\cancel{x^2} - 1 - \cancel{x^2}\right]$

$= \frac{-1}{\sqrt{1+x^2}}$

10. $(2,-4)$ to $(4,-2)$

midpoint $= (3,-3)$

slope $= \frac{-2+4}{4-2} = 1$

slope $\perp = -1$

$y - y_1 = m(x - x_1)$

$y + 3 = -1(x - 3)$

$x + y = 0$

PART II

1. midpt $BC = (6,-6)$ $\quad$ $A = (2,-4)$

a) $d = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

b) $m = \frac{-4+6}{2-6} = \frac{2}{-4} = -\frac{1}{2}$

$y + 4 = -\frac{1}{2}(x - 2)$

$2y + 8 = -x + 2$

$x + 2y + 6 = 0$

2. $\left|\frac{3-2x}{x+4}\right| < 1$ $\quad$ $x = -4$

$\frac{3-2x}{x+4} = 1$ $\quad$ $\frac{3-2x}{x+4} = -1$

$3 - 2x = x + 4$ $\quad$ $3 - 2x = -x - 4$

$-1 = 3x$ $\quad$ $7 = x$

$x = -1/3$

Endpoints: $-4, -1/3, 7$

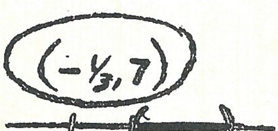
Test -5: $\left|\frac{13}{1}\right| < 1$ No

-1: $\left|\frac{5}{3}\right| < 1$ No

0: $\left|\frac{3}{4}\right| < 1$ Yes

8: $\left|\frac{-13}{12}\right| < 1$ No.

$(-1/3, 7)$



3. (See example 4, p.39)

a) $x^2 y = x - 3$

$x = 0$, $0 = -3$ (No y int.)

$y = 0$, $x = 3$ $\quad$ $(3,0)$

$y x^2 - x + 3 = 0$

$x = \frac{+1 \pm \sqrt{1 - 12y}}{2y}$

$y = \frac{x-3}{x^2}$

Note: It looks like $y \neq 0$. But in original prob. $y = 0$ gives $x = 3$. So y can equal zero.

b) Asymptotes $x = 0$, $y = 0$

c) Domain: all $x \neq 0$.

d) Range: $1 - 12y \geq 0$ $\quad$ $y \leq 1/12$

e) No Symmetry