CALCULUS I EXAM 2C Dr. RAPALJE

Show all work as necessary on separate paper. Turn in all work sheets. You may keep a copy of your answers and this test. <u>NO CALCULATORS</u> on this test only.

EVALUATE THE FOLLOWING LIMITS, or SHOW THAT (EXPLAIN WHY) THEY DO NOT EXIST: (4 each)

1. $\lim\limits_{x\to -1} \dfrac{2x+1}{x^2-3x+4} =$

2. $\lim\limits_{h\to 0} \dfrac{(x+h)^2-x^2}{h} =$

3. $\lim\limits_{x\to 0} \dfrac{2-\sqrt{4-x}}{x} =$

4. $\lim\limits_{x\to \infty} \dfrac{2x^3-2x}{5x^3-5} =$

5. $\lim\limits_{x\to 1} \dfrac{2x^3-2x}{5x^3-5} =$

6. $\lim\limits_{x\to 1} \sqrt{x-1} =$

7. $\lim\limits_{x\to 1^+} \dfrac{|x-1|}{x-1} =$

8. $\lim\limits_{x\to 0^-} \dfrac{x}{2}\sqrt{\dfrac{2}{x^2}} =$

9. $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{\sqrt{x^2-4}} =$

10. $\lim\limits_{x\to +\infty} \sqrt{x^2+x} - x =$ (Hint NOT $\infty-\infty=0$)

11. Explain why $\lim\limits_{x\to 0} \frac{1}{x^3}$ does not exist, but $\lim\limits_{x\to 0} \frac{1}{x^4}$ does exist.

In 12-13, use $f'(x) = \lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{h}$ to find the derivative of:

(8ea) 12. $f(x) = x^3+5$ 13. $f(x) = \dfrac{x}{x+2}$

(8) 14. Find the equation of the tangent line to $y = x^3-2x^2+3$ at $x=2$.

(12) 15. Find the equation of the normal line to $y = -x^2+2x+3$
a) at $x=0$ b) at $x=1$.

(8) 16. For what value(s) of c will $f(x) = \begin{cases} cx-1 & x<2 \\ cx^2 & x\geq 2 \end{cases}$ be continuous.
a) $\lim\limits_{x\to 2^-} f(x)$ b) $\lim\limits_{x\to 2^+} f(x)$

(12) 17. $f(x) = \begin{cases} \dfrac{x-1}{4x^2-4} & -3\leq x<3,\ x\neq \pm 1 \\ \dfrac{1}{(x+1)^2} & 3\leq x\leq 5 \end{cases}$
and $f(-1) = f(1) = \frac{1}{8}$

a) Find all possible points of discontinuity
b) Determine + show whether it is continuous or discontinuous at each point.
c) Indicate removable discontinuities

CALCULUS I EXAM 2C Solutions Dr. RAPALJE

1. $\lim\limits_{x\to -1} \dfrac{2x+1}{x^2-3x+4} = \dfrac{-2+1}{1+3+4} = -\dfrac{1}{8}$

2. $\lim\limits_{h\to 0} \dfrac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$

3. $\lim\limits_{x\to 0} \dfrac{2-\sqrt{4-x}}{x} = \dfrac{0}{0}$

$= \lim\limits_{x\to 0} \dfrac{(2-\sqrt{4-x})}{x}\dfrac{(2+\sqrt{4-x})}{(2+\sqrt{4-x})} = \lim\limits_{x\to 0} \dfrac{\cancel{4}-\cancel{4}+x}{x(2+\sqrt{4-x})}$

$= \lim\limits_{x\to 0} \dfrac{1}{2+\sqrt{4-x}} = \dfrac{1}{2+2} = \dfrac{1}{4}$

4. $\lim\limits_{x\to\infty} \dfrac{2x^3-2x}{5x^3-5} = \lim\limits_{x\to\infty} \dfrac{2-2/x^2}{5-5/x^3} = \dfrac{2}{5}$

5. $\lim\limits_{x\to 1} \dfrac{2x^3-2x}{5x^3-5} = \dfrac{0}{0} = \lim\limits_{x\to 1} \dfrac{2x(x^2-1)}{5(x^3-1)}$

$= \lim\limits_{x\to 1} \dfrac{2x\cancel{(x-1)}(x+1)}{5\cancel{(x-1)}(x^2+x+1)} = \dfrac{2\cdot 2}{5\cdot 3} = \dfrac{4}{15}$

6. $\lim\limits_{x\to 1} \sqrt{x-1}$ = Does not exist from the left side.

7. $\lim\limits_{x\to 1^+} \dfrac{|x-1|}{x-1} = \dfrac{x-1}{x-1}$ for $x-1 \geq 0$

$= 1$ for $x>1$

8. $\lim\limits_{x\to 0^-} \dfrac{x}{2}\sqrt{\dfrac{2}{x^2}} = \lim\limits_{x\to 0^-} \dfrac{x}{2}\cdot\dfrac{\sqrt{2}}{|x|}$

$= \lim\limits_{x\to 0^-} \dfrac{x}{|x|}\,\dfrac{\sqrt{2}}{2}$

$= -\dfrac{\sqrt{2}}{2}$ $x<0$

9. $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{\sqrt{x^2-4}} = \lim\limits_{x\to 2^+} \dfrac{(x-2)(x-1)}{\sqrt{(x-2)(x+2)}}$

$= \lim\limits_{x\to 2^+} \dfrac{(x-2)^{1/2}(x-1)}{(x+2)^{1/2}} = 0$

(NOTE: Rationalizing denom. also works!)

10. $\lim\limits_{x\to +\infty} (\sqrt{x^2+x}-x)\dfrac{(\sqrt{x^2+x}+x)}{(\sqrt{x^2+x}+x)}$

$= \lim\limits_{x\to +\infty} \dfrac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \dfrac{x}{\sqrt{x^2(1+\frac{1}{x})}+x}$

$= \lim\limits_{x\to +\infty} \dfrac{x}{|x|\sqrt{1+\frac{1}{x}}+x} = \dfrac{1}{1+1} = \dfrac{1}{2}$

11. $\left.\begin{array}{l} \lim\limits_{x\to 0^+} \dfrac{1}{x^3} = +\infty \\ \lim\limits_{x\to 0^-} \dfrac{1}{x^3} = -\infty \end{array}\right\}$ $\lim\limits_{x\to 0}$ does not exist. They are not the same.

$\left.\begin{array}{l} \lim\limits_{x\to 0^+} \dfrac{1}{x^4} = +\infty \\ \lim\limits_{x\to 0^-} \dfrac{1}{x^4} = +\infty \end{array}\right\}$ $\lim\limits_{x\to 0}$ does exist. They are same.

12. $f(x) = x^3+5$

$f(x+\Delta x) = (x+\Delta x)^3+5$

$f'(x) = \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{x^3}+3x^2\Delta x+3x\Delta x^2+\Delta x^3\cancel{+8}-\cancel{x^3}-\cancel{8}}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{\Delta x}(3x^2+3x\Delta x+\Delta x^2)}{\cancel{\Delta x}} = 3x^2$

13. $f(x) = \dfrac{x}{x+2}$ $\quad f(x+\Delta x) = \dfrac{x+\Delta x}{x+\Delta x+2}$

$f'(x) = \lim\limits_{\Delta x\to 0} \dfrac{\dfrac{x+\Delta x}{x+\Delta x+2} - \dfrac{x}{x+2}}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{x^2}+\cancel{2x}+\cancel{x\Delta x}+2\Delta x-\cancel{x^2}-\cancel{x\Delta x}-\cancel{2x}}{(x+\Delta x+2)(x+2)\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{2\cancel{\Delta x}}{\cancel{\Delta x}(x+\Delta x+2)(x+2)} = \dfrac{2}{(x+2)^2}$

14. $y = x^3-2x^2+3$

$y' = 3x^2-4x$

$y'(2) = 4$

$y(2) = 8-8+3 = 3$

$y-3 = 4(x-2)$

$y = 4x-5$

15. $y = -x^2+2x+3$

$y' = -2x+2$

$y'(0) = 2$

$y'_{\text{NORMAL}} = -\frac{1}{2}$

$y(0) = 3$

$y-3 = -\frac{1}{2}(x-0)$

a) $y = -\frac{1}{2}x+3$

b) $y'(1) = 0 \quad y'_{\perp} = \infty$

Vertical line $x = 1$

16. $f(x) = \begin{cases} cx-1 \\ cx^2 \end{cases}$

a) $\lim\limits_{x\to 2^-} = 2c-1$

b) $\lim\limits_{x\to 2^+} = c\cdot 2^2 = 4c$

C^0 if $2c-1 = 4c$

$-1 = 2c$

$c = -\frac{1}{2}$

17. a) $x = 1, -1, 3$

b) $\lim\limits_{x\to 3^+} = \frac{1}{16} = \lim\limits_{x\to 3^-} = \frac{1}{16}$ Continuous

b) $\lim\limits_{x\to 1} f(x) = \dfrac{\cancel{x-1}}{4\cancel{(x-1)}(x+1)} = \dfrac{1}{8}$

$f(1) = 1/8$ Continuous

$\lim\limits_{x\to -1} = \dfrac{1}{0} = \infty$ Asymptote

$f(-1) = 1/8$ Discont.

c) Removable at $x=1$.