CALCULUS I EXAM 2D

Dr. RAPALJE

TURN IN THIS TEST WITH ALL WORK SHEETS.

SHOW ALL WORK ON SEPARATE PAPER (NEATLY PLEASE!)

SORRY, STILL NO CALCULATORS.

1. Find $f'(x)$

20% a) $f(x) = 5 - \dfrac{5}{2x^3}$ b) $f(x) = (x^3-3x+6)\sqrt[3]{(x^3-3x+6)^2}$

2. Find $f'(x)$ and factor completely without negative exponents:

20% $f(x) = \dfrac{\sqrt{5x-2}}{(x^2+9)^4}$

3. Given the relation $x^3y + 2y^2 = -6$

   a) Find y' by implicit differentiation (in terms of x and y)

20% b) Solve for y in terms of x and select the particular function which passes through $(2,-1)$

   c) Find y' as a function of x for this function.

   d) Evaluate y' at $(2,-1)$ and compare answers.

4. Do any 4 of the following problems (Omit one yourself.)

   a) $\displaystyle\lim_{x\to 2^-} \frac{2-3x+x^2}{\sqrt{4-x^2}}$ b) $\displaystyle\lim_{x\to\infty} \frac{2x^2}{x^2-4}$ c) $\displaystyle\lim_{x\to 2} \frac{2x^2}{x^2-4}$

40% d) Find all possible points of discontinuity, determine and show whether each point is continuous or discontinuous:

$$f(x) = \begin{cases} \dfrac{2x+4}{4x^2-16} & -3 \le x < 3,\ x \ne \pm 2 \\ 9x^{-2} & 3 \le x \le 5 \end{cases}$$

$f(-2) = f(2) = 0$

   e) $\displaystyle\lim_{x\to-\infty} \left(3x + \sqrt{9x^2 - x}\right)$ (Hint: Not $-\infty + \infty = 0$)

CALCULUS I EXAM 2D Answers Dr. RAPALJE

1a) $f(x) = 5 - \frac{5}{2x^3}$

$= 5 - \frac{5}{2}x^{-3}$

$f'(x) = \frac{5}{2}\cdot 3x^{-4}$

$= \frac{15}{2x^4}$

b) $f(x) = (x^3-3x+6)^{5/3}$

$f'(x) = \frac{5}{3}(x^3-3x+6)^{2/3}(3x^2-3)$

$= 5(x^2-1)(x^3-3x+6)^{2/3}$

2. $f(x) = \frac{\sqrt{5x-2}}{(x^2+9)^4}$

$$f'(x) = \frac{(x^2+9)^4\frac{1}{2}(5x-2)^{-1/2}\cdot 5 - (5x-2)^{1/2}4(x^2+9)^3 2x}{(x^2+9)^8}$$

$$= \frac{(5x-2)^{-1/2}(x^2+9)^3\left[\frac{5}{2}(x^2+9) - 8x(5x-2)\right]}{(x^2+9)^8}$$

$$= \frac{(5x-2)^{-1/2}}{(x^2+9)^5}\,\frac{(5x^2+45-80x^2+32x)}{2}$$

$$= \frac{-75x^2+32x+45}{2(x^2+9)^5\sqrt{5x-2}}$$

3. $x^3y + 2y^2 = -6$

a) $x^3y' + y3x^2 + 4yy' = 0$

$y'(x^3+4y) = -3x^2y$

$y' = \frac{-3x^2y}{x^3+4y}$

b) $2y^2 + x^3y + 6 = 0$

$a=2 \quad b=x^3 \quad c=6$

$y = \frac{-x^3 \pm \sqrt{x^6-48}}{4}$

$y_1 = \frac{-x^3+\sqrt{x^6-48}}{4}$ $\quad y_2 = \frac{-x^3-\sqrt{x^6-48}}{4}$

$y_1(2) = \frac{-8+\sqrt{16}}{4} = \frac{-8+4}{4} = -1$ $\quad y_2(2) = \frac{-8-\sqrt{16}}{4} = \frac{-8-4}{4} = -3$

Select y_1

c) $y_1 = -\frac{1}{4}x^3 + \frac{1}{4}(x^6-48)^{1/2}$

$y_1' = -\frac{3}{4}x^2 + \frac{1}{8}(x^6-48)^{-1/2}6x^5$

d) $y_1'(2) = -3 + \frac{1}{8}\cdot\frac{1}{4}\cdot 6\cdot 32$

$= -3+6 = 3$

$y' = \frac{-3x^2y}{x^3+4y} = \frac{+12}{8-4} = \frac{12}{4} = 3$ at $(2,-1)$

4 a) $\lim_{x\to 2^-} \frac{2-3x+x^2}{\sqrt{4-x^2}} = \frac{(2-x)(1-x)}{\sqrt{2-x}\sqrt{2+x}}$

$= \frac{\sqrt{2-x}\,(1-x)}{\sqrt{2+x}} = 0$

b) $\lim_{x\to\infty} \frac{2x^2}{x^2-4} = \frac{2}{1-4/x^2} = \frac{2}{1} = 2$

c) $\lim_{x\to 2} \frac{2x^2}{x^2-4} = \frac{8}{0} = \infty$

d) Possible points: $x=-2,\ x=2,\ x=3$

DIS $\begin{cases} f(-2)=0 \\ \lim_{x\to -2} \frac{2\cancel{(x+2)}}{4(x-2)\cancel{(x+2)}} = \frac{2}{4\cdot(-4)} = -\frac{1}{8}\end{cases}$

DIS $\begin{cases} f(2)=0 \\ \lim_{x\to 2} = \text{asymptote.}\end{cases}$

DIS $\begin{cases} \lim_{x\to 3^-} \frac{2\cancel{(x+2)}}{4(x-2)\cancel{(x+2)}} = \frac{1}{2} \\ \lim_{x\to 3^+} 9x^{-2} = \frac{9}{9} = 1\end{cases}$

All three are discontinuous

4e) $\lim_{x\to-\infty} \frac{(3x+\sqrt{9x^2-x})(3x-\sqrt{9x^2-x})}{3x-\sqrt{9x^2-x}}$

$$= \frac{9x^2-(9x^2-x)}{3x-\sqrt{9x^2(1-\frac{1}{9x})}} = \frac{x}{3x-3|x|\sqrt{\ }}$$

$\lim_{x\to-\infty}\frac{x}{3x-3(-x)\sqrt{\ }}$ since $|x| = -x$ if $x<0$.

$$= \frac{x}{3x+3x\sqrt{1-\cancel{\frac{1}{9x}}^{\,0}}} = \frac{x}{6x} = \frac{1}{6}$$