CALCULUS I
EXAM 2E

[SHOW ALL WORK AS NECESSARY ON SEPARATE PAPER]

Dr. RAPALJE

1-5 Find $\frac{dy}{dx}$ (GIVE answers in factored form)

(1.) $y = \sqrt{x^3 + 6x}$

(2.) $y = (x^2+5)^4 (\sqrt[3]{x^3-5})$

(3.) $y = \frac{2x}{\sqrt{x^2+4}}$

(4.) $y = \frac{2}{3}\sin^{\frac{3}{2}} x - \frac{2}{7}\sin^{\frac{7}{2}} x$ (simplify answer)

(5.) $x^2 + y^2 - 6x + 2y = 10$

(6.) Use 4 step method to find $\frac{dy}{dx}$ for: $y = \frac{1}{x^2}$

(7.) Find 2nd derivative of $f(x) = x^4 + 3x^2 - 2x + 4 - \frac{1}{x}$

(8.) Find equation of tangent line to: $y = \frac{2x}{3-x^2}$ at $(2, -4)$

(9.) For what value(s) of x does $y = \frac{x^2}{x-1}$ have a horizontal tangent?

(10.) Given position function $S(t) = -16t^2 + 600$, where a ball is dropped from the top of a 600 foot building

a.) Find function for velocity

b.) When will the ball hit the ground?

c.) Find the velocity when it hits the ground?

(11.) Water runs into a conical tank at the rate of $2\ ft^3/min$. The tank stands point down, it has a height of 10 feet, and the base radius is 5 feet. How fast is the water level rising when the water is 6 feet deep?

(12.) A man 6 ft. tall walks at a rate of 10 ft/sec. away from a light that is 15 ft. above the ground. How fast is the length of the shadow changing when he is 10 feet from the light?

(13.) Use the four step method to show that if $y = f(x) = \sin x$ then $\frac{dy}{dx} = \cos x$

(14.) Show that if $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$

[Hint: Use $\tan X = \frac{\sin X}{\cos X}$ and quotient rule]

CALCULUS I EXAM 2E Solutions Dr. Rapalje

1. $y = (x^3+6x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^3+6x)^{-1/2}\cdot 3x^2$

$= \frac{3x^2+6}{2\sqrt{x^3+6x}}$

2. $y = (x^2+5)^4(x^3-5)^{1/3}$

$\frac{dy}{dx} = (x^2+5)^4\frac{1}{3}(x^3-5)^{-2/3}3x^2 + (x^3-5)^{1/3}4(x^2+5)^3\cdot 2x$

$= (x^2+5)^3(x^3-5)^{-2/3}\left[x^2(x^3+5)+8x(x^3-5)\right]$

$= \frac{(x^2+5)^3\left[x^4+5x^2+8x^4-40x\right]}{(x^3-5)^{2/3}}$

$= \frac{(x^2+5)^3 x(9x^3+5x-40)}{(x^3-5)^{2/3}}$

3. $y = \frac{2x}{(x^2+4)^{1/2}}$

$\frac{dy}{dx} = \frac{(x^2+4)^{1/2}\cdot 2 - 2x\frac{1}{2}(x^2+4)^{-1/2}2x}{(x^2+4)}$

$= \frac{2(x^2+4)^{-1/2}\left[(x^2+4)-x^2\right]}{(x^2+4)}$

$= \frac{2\cdot 4}{(x^2+4)^{3/2}}$ or $\frac{8}{(x^2+4)^{3/2}}$

4. $y = \frac{2}{3}\sin^{3/2}x - \frac{2}{7}\sin^{7/2}x$

$\frac{dy}{dx} = \frac{2}{3}\frac{3}{2}\sin^{1/2}x\cdot\cos x - \frac{2}{7}\frac{7}{2}\sin^{5/2}x\cos x$

$= \sin^{1/2}x\cos x(1-\sin^2 x)$

$= \sin^{1/2}x\cos x\cdot\cos^2 x$

$= \sin^{1/2}x\cos^3 x$

5. $x^2+y^2-6x+2y=10$

$2x+2y\frac{dy}{dx}-6+2\frac{dy}{dx}=0$

$(2y+2)\frac{dy}{dx} = 6-2x$

$\frac{dy}{dx} = \frac{3-x}{y+1}$

6. $f(x) = \frac{1}{x^2}$ $f(x+\Delta x) = \frac{1}{(x+\Delta x)^2}$

step 2: $f(x+\Delta x)-f(x) = \frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}$

$= \frac{x^2 - x^2 - 2x\Delta x - (\Delta x)^2}{x^2(x+\Delta x)^2}$

step 3: $\frac{-2x\Delta x-(\Delta x)^2}{x^2(x+\Delta x)^2}\cdot\frac{1}{\Delta x}$

$= \frac{\Delta x(-2x-\Delta x)}{x^2(x+\Delta x)^2}\cdot\frac{1}{\Delta x}$

step 4: $\lim_{\Delta x\to 0} = \frac{-2x}{x^2\cdot x^2} = \frac{-2}{x^3}$

7. $f(x) = x^4+3x^2-2x+4-x^{-1}$

$f'(x) = 4x^3+6x-2+x^{-2}$

$f''(x) = 12x^2+6-2x^{-3}$

$= 12x^2+6-\frac{2}{x^3}$

8. $y = \frac{2x}{3-x^2}$ $(2,-4)$

$\frac{dy}{dx} = \frac{(3-x^2)\cdot 2 - 2x(-2x)}{(3-x^2)^2}$

$= \frac{6-2x^2+4x^2}{(3-x^2)^2} = \frac{6+2x^2}{(3-x^2)^2}$

$\frac{dy}{dx}(\text{at } x=2) = \frac{14}{1} = m.$

$y-y_1 = m(x-x_1)$

$y+4 = 14(x-2)$

$y = 14x-32$

9. $y = \frac{x^2}{x-1}$

$\frac{dy}{dx} = \frac{(x-1)\cdot 2x - x^2\cdot 1}{(x-1)^2}$

$= \frac{2x^2-2x-x^2}{(x-1)^2}$

$= \frac{x^2-2x}{(x-1)^2} = 0$

at $x=0, x=2$

10. $s(t) = -16t^2+600$

a) $v(t) = -32t$

b) $s(t) = -16t^2+600 = 0$

$t^2 = \frac{600}{16}$

$t = \frac{\pm 10\sqrt{6}}{4}$

$t = \frac{5\sqrt{6}}{2}$ sec.

c) $v = -32t$

$= -32\left(\frac{5\sqrt{6}}{2}\right) = -80\sqrt{6}$ ft/sec (down)

11. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\frac{h^2}{4}h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$

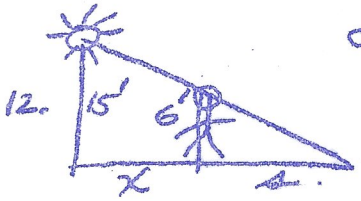
$2\text{ cu ft/min} = \frac{1}{4}\pi\cdot 3^2\text{ ft}^2\frac{dh}{dt}$

$\frac{2}{9\pi}\text{ ft/min} = \frac{dh}{dt}$

$\frac{5}{10} = \frac{r}{h}$

$5h = 10r$

$r = \frac{h}{2}$

12. 

$\frac{15}{x+s} = \frac{6}{s}$

$15s = 6x+6s$

$9s = 6x$

$9\frac{ds}{dt} = 6\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{2}{3}(10\text{ ft/sec})$

$= \frac{20}{3}\text{ ft/sec}.$

13. $f(x+\Delta x)-f(x)$

$= \sin(x+\Delta x)-\sin x$

$= \sin x\cos\Delta x+\cos x\sin\Delta x-\sin x$

$f'(x) = \lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x-\sin x+\cos x\sin\Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0}\sin x\frac{(\cos\Delta x-1)}{\Delta x} + \cos x\frac{\sin\Delta x}{\Delta x}$

$= \cos x$

14. $f(x) = \tan x = \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos x\cdot\cos x-\sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x+\sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$