CALCULUS I EXAM 3D "Derivatives" Dr. RAPALJE

In 1–5, find $f'(x)$, and simplify, factoring as necessary!

1. $f(x) = (x^2+1)^3(2x-5)^4$

2. $f(x) = \left(\dfrac{x+2}{x^2+2x-1}\right)^3$

3. $f(x) = \dfrac{1}{\sqrt[3]{x^2+2x+3}}$

4. $f(x) = (5x^2+3)\sqrt{5x^2+3}$

5. $f(x) = \sqrt[3]{x^2+2x} \cdot \sqrt{(1+2x)^3}$

6. Find the equations of the tangent and normal lines to $y = (x^2-1)^{2/3}$ at $x=3$.

7. If $\frac{y}{x} - \frac{x}{y} = 1$, a) Differentiate implicitly, to obtain $y' = \frac{y}{x}$.
b) Rewrite the equation in the form $y^2-x^2=xy$, and differentiate implicitly to obtain $y' = \dfrac{2x+y}{2y-x}$

8. If $xy^2-x^2+6=0$

a) Find y' by implicit differentiation. Evaluate at $(3,-1)$.

b) Solve the original equation for y.

c) Select the particular branch of y that passes through $(3,-1)$.

d) Find y' for the branch of y selected in part c).

e) Evaluate at $x=3$, and check with part a).

CALCULUS I EXAM 3D solutions. Dr. RAPALJE

1. $f(x) = (x^2+1)^3(2x-5)^4$

$f'(x) = (x^2+1)^3 4(2x-5)^3(2) + (2x-5)^4 \cdot 3(x^2+1)^2 2x$

$= 2(x^2+1)^2(2x-5)^3[4(x^2+1)+3x(2x-5)]$

$= 2(x^2+1)^2(2x-5)^3(10x^2-15x+4)$

2. $f(x) = \left(\dfrac{x+2}{x^2+2x-1}\right)^3$

$f'(x) = 3\left(\dfrac{x+2}{x^2+2x-1}\right)^2\left(\dfrac{(x^2+2x-1)\cdot 1-(x+2)(2x+2)}{(x^2+2x-1)^2}\right)$

$= 3\dfrac{(x+2)^2}{(x^2+2x-1)^2}\ \dfrac{x^2+2x-1-2x^2-6x-4}{(x^2+2x-1)^2}$

$= \dfrac{3(x+2)^2(-x^2-4x-5)}{(x^2+2x-1)^4}$

3. $f(x) = \dfrac{1}{\sqrt[3]{x^2+2x+3}} = (x^2+2x+3)^{-1/3}$

$f'(x) = -\frac{1}{3}(x^2+2x+3)^{-4/3}(2x+2)$

$= \dfrac{-2(x+1)}{3(x^2+2x+3)^{4/3}}$

4. $f(x) = (5x^2+3)\sqrt{5x^2+3}$

$= (5x^2+3)^{3/2}$

$f'(x) = \frac{3}{2}(5x^2+3)^{1/2}(10x) = 15x\sqrt{5x^2+3}$

5. $f(x) = \sqrt[3]{x^2+2x} \cdot \sqrt{(1+2x)^3}$

$f(x) = (x^2+2x)^{1/3}(1+2x)^{3/2}$

$f'(x) = (x^2+2x)^{1/3}\frac{3}{2}(1+2x)^{1/2}2 + (1+2x)^{3/2}\frac{1}{3}(x^2+2x)^{-2/3}(2x+2)$

$= (x^2+2x)^{-2/3}(1+2x)^{1/2}\left[3(x^2+2x)+\frac{1}{3}(1+2x)(2x+2)\right]$

$= \dfrac{(1+2x)^{1/2}}{(x^2+2x)^{2/3}}\left[\dfrac{9x^2+18x+4x^2+6x+2}{3}\right]$

$= \dfrac{(1+2x)^{1/2}(13x^2+24x+2)}{3(x^2+2x)^{2/3}}$

6. $y = (x^2-1)^{2/3}$ at $x=3$

$y' = \frac{2}{3}(x^2-1)^{-1/3}2x \qquad y=4$

$= \dfrac{4x}{3(x^2-1)^{1/3}}$

$y'(3) = \dfrac{12}{3\cdot 8^{1/3}} = 2 = m_T$

$-\frac{1}{2} = m_N$

TAN. LINE:

$y-4 = 2(x-3)$

$y-4 = 2x-6$

$2x-y-2=0$

NORMAL LINE:

$y-4 = -\frac{1}{2}(x-3)$

$2y-8 = -x+3$

$x+2y-11=0$

7. $\dfrac{y}{x} - \dfrac{x}{y} = 1$

a) $\dfrac{xy'-y\cdot 1}{x^2} - \dfrac{y\cdot 1 - x\cdot y'}{y^2} = 0$

$\dfrac{xy'-y}{x^2} = \dfrac{y-xy'}{y^2}$ Cross mult.

$xy^2y'-y^3 = x^2y-x^3y'$

$xy^2y'+x^3y' = x^2y+y^3$

$xy'(y^2+x^2) = y(x^2+y^2)$

$y' = \dfrac{y(x^2+y^2)}{x(y^2+x^2)} = \dfrac{y}{x}$

7b) $y^2-x^2 = xy$

$2yy'-2x = xy'+y\cdot 1$

$2yy'-xy' = 2x+y$

$y'(2y-x) = (2x+y)$

$y' = \dfrac{2x+y}{2y-x}$

8a) $xy^2-x^2+6=0$

$x\cdot 2yy'+y^2\cdot 1-2x=0$

$2xyy' = 2x-y^2$

$y' = \dfrac{2x-y^2}{2xy}$

$y'(3,-1) = \dfrac{6-1}{-6} = -\dfrac{5}{6}$

8b) $xy^2 = x^2-6$

$y^2 = \dfrac{x^2-6}{x}$

$y_1 = \sqrt{\dfrac{x^2-6}{x}} \qquad y_2 = -\sqrt{\dfrac{x^2-6}{x}}$

$y_1(3) = \sqrt{\dfrac{3}{3}} \qquad y_2 = -\sqrt{\dfrac{3}{3}}$

$= 1 \qquad = -1$

8c) Select $y_2 = -\sqrt{\dfrac{x^2-6}{x}}$

8d) $y_2' = -\frac{1}{2}\left(\dfrac{x^2-6}{x}\right)^{-1/2}\left(\dfrac{x\cdot 2x-(x^2-6)}{x^2}\right)$

$= -\frac{1}{2}\left(\dfrac{x}{x^2-6}\right)^{1/2}\dfrac{x^2+6}{x^2}$

8e) $y_2'(3) = -\frac{1}{2}\cdot 1\cdot \dfrac{\cancel{15}\,5}{\cancel{9}\,3}$

$= -5/6$