CALCULUS I EXAM 3C

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Show all work on separate paper. Calculators are allowed.
You may keep this test and a copy of your answers.

1. Find $\frac{dy}{dx}$: $y = 3x^3 + 2x - 4 - \frac{3}{4x^7}$

2. Find $\frac{dy}{dx}$: $y = (x^2+1)^4(x^3-3x)^5$ (FACTOR and SIMPLIFY!)

3. Find $\frac{dy}{dx}$: $y = (x^2+2x+3)^2\sqrt{x^2+2x+3}$

4. Find $\frac{dy}{dx}$: $y = (2x+3)^4(3x-2)^{7/2}$

5. Find $\frac{dy}{dx}$: $y = \frac{\sqrt[3]{t^2+2t}}{t^2}$

6. Find $\frac{dy}{dx}$: $y = \sqrt{1+\cos(x^2)}$

7. Find $\frac{dy}{dx}$: $x^2 + 2xy + 2y^2 + 3x - y = 9$

8. Find the equations of the normal line and tangent line to $16x^4 + y^4 = 32$ at $(1,2)$.

9. Calculate an approximate value using differentials:
$$(1.01)^2 + (1.01)^4 + (1.01)^6$$

10. Use $f'(x) = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ to show that if $f(x) = \sin x$, then $f'(x) = \cos x$.

11. Use $\cot x = \frac{\cos x}{\sin x}$ and the quotient rule to show that if $f(x) = \cot x$, then $f'(x) = -\csc^2 x$.

12. Find the second and third derivatives of $y = \tan x$.

CALCULUS I Exam 3C Solutions Dr. RAPALJE

1. $y = 3x^3 + 2x - 4 - \frac{3}{4}x^{-7}$

$\frac{dy}{dx} = 9x^2 + 2 + \frac{21}{4}x^{-8}$

or $9x^2 + 2 + \frac{21}{4x^8}$

2. $y = (x^2+1)^4(x^3-3x)^5$

$\frac{dy}{dx} = (x^2+1)^4 5(x^3-3x)^4(3x^2-3) + (x^3-3x)^5 4(x^2+1)^3(2x)$

$= (x^2+1)^3(x^3-3x)^4[(15x^2-15)(x^2+1) + 8x(x^3-3x)]$

$= (x^2+1)^3(x^3-3x)^4[15x^4 - 15 + 8x^4 - 24x^2]$

$= (x^2+1)^3(x^3-3x)^4(23x^4 - 24x^2 - 15)$

3. $y = (x^2+2x+3)^2\sqrt{x^2+2x+3}$

$y = (x^2+2x+3)^{5/2}$

$\frac{dy}{dx} = \frac{5}{2}(x^2+2x+3)^{3/2}(2x+2)$

$= 5(x+1)(x^2+2x+3)^{3/2}$

4. $y = (2x+3)^4(3x-2)^{7/2}$

$\frac{dy}{dx} = (2x+3)^4\frac{7}{2}(3x-2)^{5/2}\cdot 3 + (3x-2)^{7/2}4(2x+3)^3\cdot 2$

$= (2x+3)^3(3x-2)^{5/2}\left[\frac{21}{2}(2x+3) + \frac{16}{2}(3x-2)\right]$

$= (2x+3)^3(3x-2)^{5/2}\left[\frac{42x + 63 + 48x - 32}{2}\right]$

$= (2x+3)^3(3x-2)^{5/2}\left[\frac{90x+31}{2}\right]$

5. $y = \frac{\sqrt[3]{t^2+2t}}{t^2}$

$\frac{dy}{dt} = \frac{t^2\frac{1}{3}(t^2+2t)^{-2/3}(2t+2) - (t^2+2t)^{1/3}2t}{t^4}$

$= \frac{(t^2+2t)^{-2/3}2t\left[\frac{1}{3}t(t+1) - \frac{3}{3}(t^2+2t)\right]}{t^4}$

$= \frac{2(t^2+t-3t^2-6t)}{3t^3(t^2+2t)^{2/3}} = \frac{2(-2t^2-5t)}{3t^3(t^3+2t)^{2/3}}$

$= \frac{-2(2t+5)}{3t^2(t^3+2t)^{2/3}}$

6. $y = \sqrt{1+\cos(x^2)}$

$\frac{dy}{dx} = \frac{1}{2}(1+\cos(x^2))^{-1/2}(-\sin(x^2))\cdot 2x$

$= \frac{-x\sin(x^2)}{\sqrt{1+\cos(x^2)}}$

7. $x^2 + 2xy + 2y^2 + 3x - y = 9$

$2x + 2x\frac{dy}{dx} + y\cdot 2 + 4y\frac{dy}{dx} + 3 - \frac{dy}{dx} = 0$

$2x + 2y + 3 = \frac{dy}{dx}(-2x - 4y + 1)$

$\frac{dy}{dx} = \frac{2x+2y+3}{-2x-4y+1}$

8. $16x^4 + y^4 = 32$ at $(1,2)$

$64x^3 + 4y^3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{16x^3}{y^3} = \frac{-16}{8} = -2 = m_T$

$m_N = \frac{1}{2}$

TANGENT LINE

$y - 2 = -2(x-1)$

$y - 2 = -2x + 2$

$y = -2x + 4$

NORMAL LINE

$y - 2 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x + \frac{3}{2}$

9. $f(x) = x^2 + x^4 + x^6$ $x=1$ $\Delta x = .01$

$f'(x) = 2x + 4x^3 + 6x^5$

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

$= 3 + 12(.01) = 3.12$

10. $f(x) = \sin x$

$f(x+\Delta x) = \sin(x+\Delta x) = \sin x\cos\Delta x + \sin\Delta x\cos x$

$f'(x) = \lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x + \sin\Delta x\cos x - \sin x}{\Delta x}$

$= \lim_{\Delta x\to 0}\frac{\sin x(\cos\Delta x - 1)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\sin\Delta x\cos x}{\Delta x}$

$= \sin x\lim_{\Delta x\to 0}\frac{\cos\Delta x - 1}{\Delta x} + \cos x\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}$

$= \sin x\cdot 0 + \cos x\cdot 1 = \cos x$

11. $f(x) = \cot x = \frac{\cos x}{\sin x}$

$f'(x) = \frac{(\sin x)(-\sin x) - \cos x\cos x}{\sin^2 x}$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= -\frac{1}{\sin^2 x} = -\csc^2 x$

12. $y = \tan x$

$y' = \sec^2 x$

$y'' = 2\sec x(\sec x\tan x)$

$= 2\sec^2 x\tan x$

$y''' = 2\sec^2 x(\sec^2 x) + \tan x\, 4\sec x(\sec x\tan x)$

$= 2\sec^4 x + 4\sec^2 x\tan^2 x$ or $2\sec^2 x(\sec^2 x + 2\tan^2 x)$