CALCULUS I   EXAM 4C Chapter 4   Dr. RAPALJE

Show all work as necessary on separate paper. (12 points each)
You may keep this test and a copy of your answers.

1. If $y = x\sqrt{25-x^2}$, find y' and determine all critical values of x.

2. Given the table:

$\lim_{x\to-\infty} = 0$

$\lim_{x\to+\infty} = -\infty$

x		-2		-1		0		1	
y		∞		1		0		1	
y'	$-$	∞	$-$	$-$	$-$	∞	$+$	0	$-$
y''	$-$	∞	$+$	0	$-$	∞	$-$	$-$	$-$

a) Graph
b) Identify all critical points, vertical tangents, vert. asymptotes, points of inflection.

3. Given $y = x^{4/3} + 4x^{1/3}$ or $x^{1/3}(x+4)$

$y' = \dfrac{4(x+1)}{3x^{2/3}}$

$y'' = \dfrac{4(x-2)}{9x^{5/3}}$

a) Make an appropriate table as in #2.
b) Give intervals of concave up/down, and intervals of increasing/decreasing.
c) Local max, local minimums?
d) Graph.

4a) Does the Theorem of the Mean $\left[f'(c) = \dfrac{f(b)-f(a)}{b-a}\right]$ hold for $f(x) = \dfrac{3x+2}{3x-2}$ for $[1,5]$?
b) Does it apply for $[0,5]$?
c) If the Theorem holds for the above, find all values of c for which it is satisfied?
d) If the Theorem does not apply, does this mean that no value of c can be found?

5. Find the maximum and minimum values of $f(x) = \dfrac{x^2}{2x-4}$ a) for $[3,7]$; b) $[-1,7]$ (if they exist!)

6. Sand is dropped on a conical pile at 15 m³/min. The height is always equal to the diameter of the pile. How fast is the height changing when the pile is 8 m. high?

7. The sum of one number and 3 times another number is 60. Find the two numbers which would have the maximum product.

8. A cylindrical can is to have a volume of 27 cm³. If top and bottom are to be cut from squares and the residue wasted, find the dimensions of the can so that the total amount of metal used, including waste, is a minimum.

CALCULUS I Exam 4C Solutions Dr. RAPALJE

1. $y = x\sqrt{25-x^2}$ D: $-5 \le x \le 5$

$y' = x\,\frac{1}{2}(25-x^2)^{-1/2}(-2x) + (25-x^2)^{1/2}\cdot 1$

$= (25-x^2)^{-1/2}\left[-x^2 + (25-x^2)\right]$

$= \dfrac{25-2x^2}{\sqrt{25-x^2}}$

Critical point: f must be defined and C^0 on $[a,b]$ and x_0 in (a,b). x_0 is a critical point if $f'(x_0)=0$ or $f'(x_0)$ does not exist.

$y'=0$ at $x = \pm\frac{5}{\sqrt{2}}$

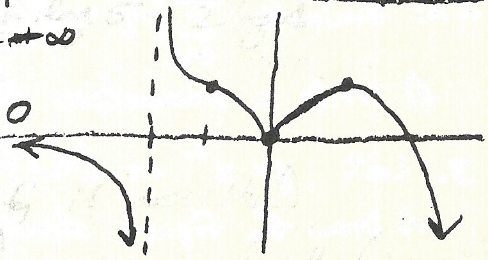
$y'=\infty$ at $x=\pm 5$, but endpoints are not critical pts (see p. 208)

2.

x		-2		-1		0		1	
y	//////	∞	//////	1	//////	0	//////	1	//////
y'	$-$	∞	$-$	$-$	$-$	∞	$+$	0	$-$
y''	$-$	∞	$+$	0	$-$	∞	$-$	$-$	$-$

a) $\lim\limits_{x\to+\infty} = -\infty$

$\lim\limits_{x\to-\infty} = 0$



b) Critical points: $(0,0)$ $(1,1)$

Vert. tangents: $x=0$.

Vert asymp: $x=-2$

Inflection: $(-1,1)$

3. $y = x^{1/3}(x+4)$

$y' = \dfrac{4(x+1)}{3x^{2/3}}$

$y'' = \dfrac{4(x-2)}{9x^{5/3}}$

a)

x		-1		0		2	
y	//////	-3	//////	0	//////	$6\sqrt[3]{2}$	//////
y'	$-$	0	$+$	∞	$+$	$+$	$+$
y''	$+$	$+$	$+$	∞	$-$	0	$+$

$y'=0$ at $x=-1$ $y''=0$ at $x=2$

$y'=\infty$ at $x=0$ $y''=\infty$ at $x=0$

b) Up: $(-\infty,0)\cup(2,\infty)$

Down: $(0,2)$

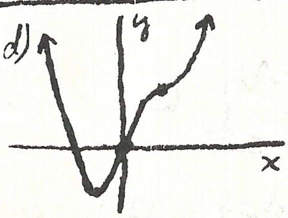
Increasing: $(-1,\infty)$

Decreasing: $(-\infty,-1)$

c) Local min: $(-1,-3)$

No Local max.

d)



4. $f(x) = \dfrac{3x+2}{3x-2}$ is continuous for all $x \ne 2/3$

$f'(x)$ exists for $x \ne 2/3$.

Theorem of Mean applies for a) $[1,5]$ but not for b) $[0,5]$

c) $f'(x) = \dfrac{(3x-2)(3)-(3x+2)3}{(3x-2)^2} = \dfrac{-12}{(3x-2)^2}$

$f'(c) = \dfrac{-12}{(3c-2)^2}$ $a=1$ $f(a)=5$, $b=5$ $f(b)=\frac{17}{13}$

$\dfrac{f(b)-f(a)}{b-a} = \dfrac{\frac{17}{13}-5}{5-1} = \dfrac{-\frac{48}{13}}{4} = -\dfrac{12}{13} = \dfrac{-12}{(3c-2)^2}$

$(3c-2)^2 = 13$

$3c-2 = \pm\sqrt{13}$

$c = \dfrac{2\pm\sqrt{13}}{3}$ $\dfrac{2+\sqrt{13}}{2}$ is in (a,b)

d) Not necessarily. It only removes the guarantee.

5. $f(x) = \dfrac{x^2}{2x-4}$ asymp at $x=2$.

$f'(x) = \dfrac{(2x-4)2x - x^2(2)}{(2x-4)^2}$

$= \dfrac{2x[2x-4-x]}{4(x-2)^2}$

$= \dfrac{x(x-4)}{2(x-2)^2} = 0$ at $x=0,4$

a) $[3,7]$: $f(3) = \frac{9}{2} = 4.5$

$f(7) = \frac{49}{10} = 4.9$ MAX

$f(4) = \frac{16}{4} = 4$ MIN

b) $[-1,7]$: $\lim\limits_{x\to2^+} = +\infty$, $\lim\limits_{x\to2^-} = -\infty$ } No MAX or MIN.

6. $V = \frac{1}{3}\pi r^2 h$ $h = 2r$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2\cdot h$ $r = \frac{h}{2}$

$V = \frac{1}{12}\pi h^3 \to \frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$

$\frac{dV}{dt} = 15\,\text{m}^3/\text{min}$, $h=8$

$15 = \frac{1}{4}\pi 64\cdot\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{15}{16\pi}$ m/min.

7. $x + 3y = 60$

Minimize $P = xy$

$P = (60-3y)y$

$P = 60y - 3y^2$

$P' = 60 - 6y = 0$

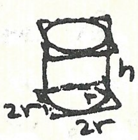
$y = 10$

$x = 30$

$P'' = -6$ Concave down

∴ Maximum product

8.



Metal $= 2\pi rh + 2(2r)^2$

$= 2\pi rh + 8r^2$

$V = \pi r^2 h = 27\,\text{cm}^3$

Minimize metal $= 2\pi r\left(\frac{27}{\pi r^2}\right) + 8r^2$

$M = 54r^{-1} + 8r^2$

$M' = -54r^{-2} + 16r = 0$

$16r = 54r^{-2}$

$r^3 = \frac{54}{16} = \frac{27}{8}$

$r = \frac{3}{2}$

$h = \frac{27}{\pi\,9/4}$

$h = \frac{12}{\pi}$

$M'' = 108r^{-3} + 16$

$M'' > 0$