CALCULUS I EXAM 5D Dr. RAPALJE

1. If $y = x\sqrt{4-x^2}$, find all values of x where

(20) $y' = 0$ or $\nexists$, $y'' = 0$ or $\nexists$.

2. If $f(x) = \frac{x^2}{x^2-4}$, $f'(x) = \frac{-8x}{(x^2-4)^2}$ and $f''(x) = \frac{24x^2+32}{(x^2-4)^3}$,

optional (a) Find $\lim_{x\to-\infty} f(x)$, $\lim_{x\to+\infty} f(x)$, $\lim_{x\to 2^+} f(x)$, $\lim_{x\to -2^+} f(x)$; $\lim_{x\to -2^-} f(x)$.

b) Make a table as in class.

c) Find all relative minimums; relative maximums.

d) Find all critical values.

(30) e) Find all points of inflection.

f) Find all intervals in which graph is concave down

g) Find all intervals in which graph is increasing.

h) Find all vertical asymptotes.

i) Find all vertical tangents.

j) Graph it.

3. If $f(x) = \frac{x}{1+x}$, $x=0$, $h=.1$, find Δf and df.

(12)

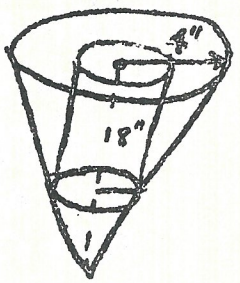
(WORST PROBLEM WILL BE OMITTED!)

4. Find two numbers x and y such that their sum is 60

(13) and the product xy^3 is a maximum.

5. Find the volume of the largest right circular cylinder that can be inscribed in a right circular cone having radius of 4 in and height of 8 in. (Hint: similar triangles!)

(13)



6. A 15' ladder leans against the wall of a house. If the bottom of the ladder is pulled horizontally away at 4 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom is 9' from the wall?

(13)

7. Sand is falling onto a sand pile at the rate of $3\,ft^3/min$, (the sandpile is conical). The base diameter is always 3 time. the altitude. At what rate is the altitude increasing when the altitude is 4 ft?

(13)

CALCULUS I EXAM 5D Solutions Dr. RAPALJE

1. $y = x\sqrt{4-x^2}$

$y' = x\,\frac{1}{2}(4-x^2)^{-1/2}(-2x) + (4-x^2)^{1/2}$

$= (4-x^2)^{-1/2}\left[-x^2 + 4 - x^2\right]$

$= \frac{4-2x^2}{(4-x^2)^{1/2}} = \frac{2(2-x^2)}{(4-x^2)^{1/2}}$

$y'' = \frac{(4-x^2)^{1/2}(-4x) - (4-2x^2)\frac{1}{2}(4-x^2)^{-1/2}(-2x)}{(4-x^2)}$

$= \frac{(4-x^2)^{-1/2}\left[(4-x^2)(-4x) + x(4-2x^2)\right]}{(4-x^2)}$

$= \frac{-16x + 4x^3 + 4x - 2x^3}{(4-x^2)^{3/2}} = \frac{2x^3 - 12x}{(4-x^2)^{3/2}}$

$\frac{2x(x^2-6)}{(4-x^2)^{3/2}}$

$y' = 0$ at $x = \pm\sqrt{2}$; $y' = *$ at $x = \pm 2$

$y'' = 0$ at $x = 0, \pm\sqrt{6}$; $y'' = *$ at $x = \pm 2$

2. $f(x) = \frac{x^2}{x^2-4}$ $f'(x) = \frac{-8x}{(x^2-4)^2}$ $f''(x) = \frac{24x^2+32}{(x^2-4)^3}$

a) $\lim_{x\to-\infty} f(x) = \frac{1}{1-4/x^2} = 1$ $\lim_{x\to+\infty} f(x) = 1$

$\lim_{x\to 2^+} f(x) = \frac{4}{+0} = +\infty$ $\lim_{x\to 2^-} f(x) = \frac{4}{-0} = -\infty$

$\lim_{x\to -2^+} f(x) = \frac{4}{-0} = -\infty$ $\lim_{x\to -2^-} f(x) = \frac{4}{+0} = +\infty$

b)

x	-3	-2	-1	0	1	2	3
f		*		0		*	
f'	+	*	+	0	−	*	−
f''	+	*	−	−	−	*	+

c) Rel min: None. Rel max: (0,0)

d) Critical points: (0,0) $f' = 0$

e) Pt of inflection: None

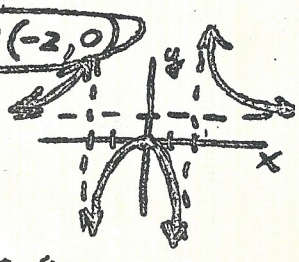
f) Concave down: $(-2, 2)$

g) Increasing: $(-\infty, -2) \cup (-2, 0)$

h) Asymp: $x = \pm 2$

i) Vert tan: None.

j)



3. $f(x) = \frac{x}{1+x}$ $x = 0$, $h = .1$

$f'(x) = \frac{(1+x) - x}{(1+x)^2} = \frac{1}{(1+x)^2}$ $f(x+h) = f(.1) = \frac{.1}{1.1} = \frac{1}{11}$

$df = f'(x)\cdot h = 1\cdot(.1) = .1$ $\Delta f = f(x+h) - f(x) = \frac{1}{11} - 0 = \frac{1}{11} = .091$

4. $x + y = 60 \Rightarrow x = 60 - y$

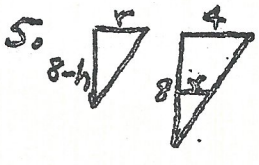
Maximize $P = xy^3 = (60-y)y^3 = 60y^3 - y^4$

$P' = 180y^2 - 4y^3 = 0$

$4y^2(45-y) = 0$

$y = 0$, $x = 60$ (Minimum) $y = 45$, $x = 15$ maximum.

5.



$\frac{4}{8} = \frac{r}{8-h}$

$\frac{1}{2} = \frac{r}{8-h}$

$2r = 8 - h$

Maximize $V = \pi r^2 h$

$V = \pi r^2(8-2r)$

$= 8\pi r^2 - 2\pi r^3$

$V' = 16\pi r - 6\pi r^2 = 0$

$2\pi r(8-3r) = 0$

$r = 0$ $r = \frac{8}{3}$

$h = 8 - 16/3 = \frac{8}{3}$

Max $V = \pi r^2 h = \pi\left(\frac{8}{3}\right)^2\left(\frac{8}{3}\right) = \frac{512\pi}{27}$

6.

$\frac{dx}{dt} = 4$ ft/sec.

$x^2 + y^2 = 15^2$

$2x\,dx + 2y\,dy = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

Now, when $x = 9$,

$81 + y^2 = 225$

$y^2 = 144$

$y = 12$.

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$9(4) + 12\frac{dy}{dt} = 0$

$12\frac{dy}{dt} = -36$, $\frac{dy}{dt} = -3$ ft/sec

7.

$\frac{dV}{dt} = 3$ ft^3/min.

$d = 3h = 2r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h$

$V = \frac{9\pi}{12}h^3 = \frac{3\pi}{4}h^3$

$\frac{dV}{dt} = \frac{9\pi h^2}{4}\frac{dh}{dt}$. At $h = 4$

$3 = \frac{9\pi\cdot 16}{4}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{12\pi}$ ft/min.