CALCULUS I EXAM 5C Calculators and Computers Allowed

Complete the first 11 problems, turn in, and then complete #12 using the Compucalc Program. One problem is to be omitted, but not #3.

1. Find y containing $(1,5)$ if $\frac{dy}{dx} = x^3 + x^2 - 3$

2 a) Evaluate $\sum_{i=2}^{20} \left(\frac{1}{i} - \frac{1}{i+1}\right)$ [Hint: find a shortcut]

b) Write a summation for $x^3 - x^6 + x^9 - x^{12} + x^{15} - x^{18} + x^{21} - x^{24}$

3. Let $y = x^2$ on $[0,2]$ Subdivide into 4 subintervals.

a) Draw the figure showing the minimum s and the maximum S.

b) Compute s.

c) Compute S.

d) Set up S (or s) for n subintervals.

e) Compute $\lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\,\Delta x$.

4. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + 2\frac{i}{n}\right)^3 \left(\frac{2}{n}\right) =$

5. $\int_{16}^{25} \frac{x-1}{\sqrt{x}}\,dx =$

6. $\int_0^2 x^2\sqrt{1+x^3}\,dx =$

7. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$

8. Set up ONLY to find the area between the curves $y = x - 5$ and $2x = y^2 + 2$. DO NOT SOLVE !!

9. Find the area between $y = x^2$ and $y = -x^2 + 4x$.

10. The density of a tree is $\rho(h) = \frac{50}{\sqrt{h+1}}$ in kg/meter, where h is the height in meters. The tree is 24 m tall.

a) Find the total mass of the tree.

b) For what value of h is half the mass of the tree above and half below?

11. A ball is thrown downward with initial velocity 48 ft/sec. from a tower 640 ft high. Beginning with acceleration $a = 32$, find formulas for velocity v and position s. How long does it take to reach the ground? How fast is it traveling when it hits the ground?

12. Use the computer to solve:
Let $f(x) = 2x^3 + x^3$ over $[1,5]$ for n subintervals.

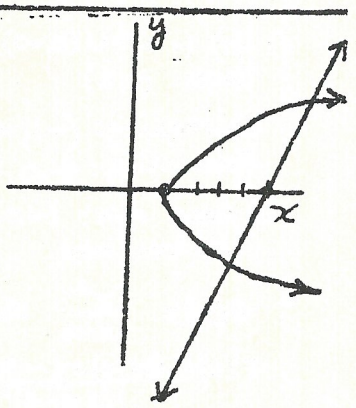
n	s	S
4		
16		
32		
$\lim_{n\to\infty}$		

FORMULAS

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$



CALCULUS I EXAM 5C Solutions Not Guaranteed !! Dr RAPALJE 7/22/16

1. $\frac{dy}{dx} = x^3 + x^2 - 3$ (1.5)

$y = \frac{x^4}{4} + \frac{x^3}{3} - 3x + C$

$5 = \frac{1}{4} + \frac{1}{3} - 3 + C$

$C = 8 - \frac{7}{12} = \frac{89}{12}$

$y = \frac{x^4}{4} + \frac{x^3}{3} - 3x + \frac{89}{12}$

2a $\sum_{i=2}^{20} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

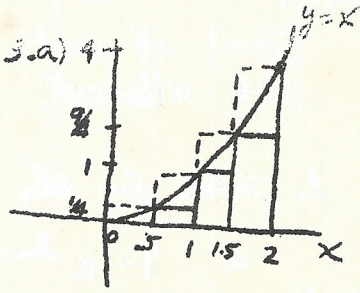
$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots \left(\frac{1}{20} - \frac{1}{21}\right)$

$= \frac{1}{2} - \frac{1}{21} = \frac{21-2}{42} = \frac{19}{42}$

b) $x^3 - x^6 + x^9 - x^{12} + x^{15} \ldots - x^{24}$

$\sum_{i=1}^{8} (-1)^{i+1} x^{3i}$ or $\sum_{i=0}^{} (-1)^i x^{3i+3}$

3a)



b) $A = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{9}{4}$

$= \frac{1}{2}\left(\frac{1}{4} + 1 + \frac{9}{4}\right) = \frac{1}{2} \cdot \frac{7}{2} = \frac{7}{4}$

c) $A = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{9}{4} + \frac{1}{2}4$

$= \frac{1}{2}\left(\frac{1}{4} + 1 + \frac{9}{4} + 2\right) = \frac{15}{4}$

3d) $\sum_{i=1}^{n} f(x_i)\,\Delta x$ $\Delta x = \frac{2}{n}$ $x_i = \frac{2i}{n}$

$= \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{2}{n}$

$= \frac{8}{n^3} \sum_{i=1}^{n} i^2$

$= \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6}$

$= \frac{4(n+1)(2n+1)}{3n^2}$

e) $\lim_{n\to\infty} \frac{4(n+1)(2n+1)}{3\cdot n \cdot n} = \frac{8}{3}$

4. $\sum_{i=1}^{n} \frac{2}{n}\left(1 + 6\frac{i}{n} + 12\frac{i^2}{n^2} + \frac{8i^3}{n^3}\right)$

$= \frac{2}{n}\sum 1 + \frac{12}{n^2}\sum i + \frac{24}{n^3}\sum i^2 + \frac{16}{n^4}\sum i^3$

$= \frac{2}{n}\cdot n + \frac{12}{n^2}\cdot\frac{n(n+1)}{2} + \frac{24}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4}\left(\frac{n(n+1)}{2}\right)^2$

$\lim_{n\to\infty} = 2 + 6 + 8 + 4 = 20$

5. $\int_{16}^{25} \frac{x-1}{\sqrt{x}}\,dx = \int_{16}^{25} (x^{1/2} - x^{-1/2})\,dx$

$= \frac{2}{3}x^{3/2} - \frac{2}{1}x^{1/2}\Big|_{16}^{25}$

$= \frac{2}{3}(125 - 64) - \frac{2}{1}(5 - 4)$

$= \frac{2}{3}(61) - 2 = \frac{122}{3} - \frac{6}{3} = \frac{116}{3}$

6. $\int_0^2 x^2\sqrt{1+x^3}\,dx$ $u = 1 + x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\int u^{1/2}\frac{du}{3}$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}(1+x^3)^{3/2}\Big|_0^2$

$= \frac{2}{9}(27 - 1) = \frac{2}{9}\cdot 26 = \frac{52}{9}$

7. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ $u = \sqrt{x}$

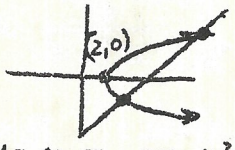
$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$= \int \cos u\, 2\,du$

$= 2\sin\sqrt{x} + C$

8.



$y = x - 5$ $2x = y^2 + 2$

$2x = (x-5)^2 + 2$

$2x = x^2 - 10x + 25 + 2$

$x^2 - 12x + 27 = 0$

$(x-3)(x-9) = 0$

$x = 3$ $y = -2$ $x = 9$ $y = 4$

Vertical rectangles: $y = \pm\sqrt{2x-2}$

$\int_2^3 \left[\sqrt{2x-2} - (-\sqrt{2x-2})\right]dx + \int_3^9 \left[\sqrt{2x-2} - (x-5)\right]dx$

Horizontal rectangles: $x = y + 5$ $x = \frac{y^2+2}{2}$

$\int_{-2}^{4} \left[(y+5) - \left(\frac{y^2+2}{2}\right)\right]dy$

9. $y = x^2$; $y = -x^2 + 4x$

$x^2 = -x^2 + 4x$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0$ $x = 2$

$\int_0^2 (-x^2 + 4x) - x^2\,dx$

$= \int_0^2 (-2x^2 + 4x)\,dx$

$= -\frac{2x^3}{3} + 2x^2\Big|_0^2$

$= -\frac{16}{3} + 8 = \frac{8}{3}$

10. $\int_0^{24} \frac{50}{\sqrt{h+1}}\,dh$ $u = h+1$

$du = dh$

$= \int 50u^{-1/2}\,du$

$= 50\cdot\frac{2}{1}u^{1/2} + C$

$= 100(h+1)^{1/2}\Big|_0^{24}$

$= 100(5-1) = 400$ kg

a) $\frac{1}{2}$ mass $= 200$ kg.

$\int_0^{h_0} \frac{50}{\sqrt{h+1}}\,dh = 200$

$100(h+1)^{1/2}\Big|_0^{h_0} = 200$

$(h_0+1)^{1/2} - 1 = 2$

$(h_0+1)^{1/2} = 3$

$h_0 + 1 = 9$

$h_0 = 8$

11. $a = 32$ $V(0) = 48$ $s(0) = 0$

$V = 32t + C_1$ $48 = 0 + C_1$

$V = 32t + 48$ ft/sec.

$s = 16t^2 + 48t + C_2$ $(C_2 = 0)$

$s = 16t^2 + 48t = 640$ ft.

$16t^2 + 48t - 640 = 0$

$16(t^2 + 3t - 40) = 0$

$(t+8)(t-5) = 0$

$t = -8$ $t = 5$ sec.

$V(5) = 32(5) + 48 = 208$ ft/sec

12. f(x) = 2*x^2+x^3
a=1, b=5
(Riemann Sums)
n=4 A=160 B=332
n=16 A=217.625
B=260.625
n=32 A=228.03125
B=249.53125
$\lim_{n\to\infty}$ = 238.666667
for a and B.