CALCULUS I EXAM 4E

Section 4.7 – 5.2 (Larson + Hostetler) DR. RAPALJE

Show all work as necessary. Calculators, approved formula sheets allowed.

In 1–6, evaluate the indefinite integral.

1. $\int \frac{x^2+6x^4+8}{4x^2}\,dx$

2. $\int (2t^2+3)^2\,dt$

3. $\int (2t^2+3)^2 t\,dt$

4. $\int \frac{\cos x}{\sin^3 x}\,dx$

5. $\int x^2\sqrt[5]{x^3+4}\,dx$

6. $\int \tan^2 y \sec^2 y\,dy$

7. Find y if $\frac{dy}{dx} = -\frac{1}{x^2}$ and the graph passes through $(1, 3)$

8. Find $y = f(x)$ if $f''(x) = x^{-1/2}$, $f'(1) = 5$, and $f(9) = -4$.

9. A ball is thrown upward from the ground with an initial velocity of 128 ft/sec. Begin with $a(t) = -32$ (constant due to gravity) and derive velocity and position formulas $v(t)$ and $s(t)$. How high will the ball go?

10. The diameter of a sphere is 18 inches with maximum error .01 in. Use the differential to estimate the possible error in the volume and surface area. Find the relative error $\frac{dV}{V}$ of volume

11. The sum of two positive numbers is 60. Find the numbers such that the product of one of the numbers times the square of the other is a minimum or maximum. Give the minimum and maximum products that can be obtained.

12. The combined perimeter of a circle and square is L (a constant). Find the dimensions of the circle and square that minimize the total area.

CALCULUS I EXAM 1 Solutions

1. $\int \frac{x^2+6x^4+8}{4x^2}\,dx$

$= \int \left(\frac{1}{4} + \frac{3x^2}{2} + 2x^{-2}\right)dx$

$= \boxed{\frac{1}{4}x + \frac{x^3}{2} - \frac{2}{x} + C}$

2. $\int (2t^2+3)^2\,dt$

$= \int (4t^4 + 12t^2 + 9)\,dt$

$= \boxed{\frac{4}{5}t^5 + 4t^3 + 9t + C}$

3. $\int (2t^2+3)^2 t\,dt$ Let $u = 2t^2+3$, $du = 4t\,dt$

$= \int u^2 \frac{du}{4}$

$= \frac{u^3}{12} = \boxed{\frac{(2t^2+3)^3}{12} + C}$

4. $\int \frac{\cos x}{\sin^3 x}\,dx$ Let $u = \sin x$, $du = \cos x\,dx$

$= \int \frac{du}{u^3} = \int u^{-3}\,du = \frac{u^{-2}}{-2}$

$= -\frac{1}{2}\sin^{-2}x + C = \boxed{-\frac{1}{2}\csc^2 x + C}$

5. $\int x^2 \sqrt[5]{x^3+4}\,dx$ Let $u = x^3+4$, $du = 3x^2\,dx$, $\frac{du}{3} = x^2\,dx$

$= \int u^{1/5}\frac{du}{3}$

$= \frac{1}{3}\frac{5}{6}u^{6/5} + C$

$= \boxed{\frac{5}{18}(x^3+4)^{6/5} + C}$

6. $\int \tan^2 y \sec^2 y\,dy$ Let $u = \tan y$, $du = \sec^2 y\,dy$

$= \int u^2\,du$

$= \frac{1}{3}u^3 = \boxed{\frac{1}{3}\tan^3 y + C}$

7. $\frac{dy}{dx} = -x^{-2}$, $(1,3)$

$y = \int -x^{-2}\,dx$

$= \frac{-x^{-1}}{-1} + C$

$y = \frac{1}{x} + C$

$3 = 1 + C$, $\boxed{y = \frac{1}{x} + 2}$

8. $f''(x) = x^{-1/2}$

$f'(x) = \frac{2}{1}x^{1/2} + C_1$

$5 = 2 + C_1$

$C_1 = 3$

$f'(x) = 2x^{1/2} + 3$

$f(x) = 2\frac{2}{3}x^{3/2} + 3x + C_2$

$-4 = \frac{4}{3}\cdot 9^{3/2} + 27 + C_2$

$-4 = 36 + 27 + C_2$

$C_2 = -67$

$\boxed{f(x) = \frac{4}{3}x^{3/2} + 3x - 67}$

9. $a(t) = -32$

$v(t) = -32t + c_1$

$v(0) = v_0 \Rightarrow c_1 = v_0$

$\boxed{v(t) = -32t + v_0}$

$s(t) = -16t^2 + v_0 t + c_2$

$s(0) = s_0 \Rightarrow c_2 = s_0$

$\boxed{s(t) = -16t^2 + v_0 t + s_0}$

$v = -32t + 128$

v at top $= 0$, so $0 = -32t + 128$

$32t = 128$

$t = 4$ sec.

$s = -16t^2 + 128t$

$= -16\cdot 4^2 + 128\cdot 4 =$

$= -256 + 512 = \boxed{256 \text{ ft}}$

10. $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2\,dr$

$= 4\pi(81)(.005)$

$= \boxed{1.62\pi \text{ cu in}}$

$S = 4\pi r^2$

$dS = 8\pi r\,dr$

$= 8\pi(9)(.005)$

$= \boxed{.36\pi \text{ sq. in}}$

$V = \frac{4}{3}\pi \cdot 9^3$

$\frac{dV}{V} = \frac{1.62\pi}{972\pi}$

$= \boxed{\frac{1}{600}}$

11. $x + y = 60$ $\quad x = 60 - y$

$P = xy^2$

$P = (60-y)y^2$

$P = 60y^2 - y^3$

$P' = 120y - 3y^2 = 0$

$3y(40-y) = 0$

$y = 0$ $\quad y = 40$

$x = 60$ $\quad x = 20$

$P = 0$ Min. $\quad P = 20\cdot 40^2$

Not Positive $\quad \boxed{= 32000 \text{ Max}}$

(Min. does not exist but lim would be zero.)

12. $2\pi r + 4s = L$

$s = \frac{1}{4}(L - 2\pi r)$

Minimize Area $= \pi r^2 + s^2$

$A = \pi r^2 + \frac{1}{16}(L - 2\pi r)^2$

$A' = 2\pi r + \frac{1}{8}(L - 2\pi r)(-2\pi) = 0$

$8\pi r - \pi L + 2\pi^2 r = 0$

$(8\pi + 2\pi^2)r = \pi L$

$\boxed{r = \frac{L}{8+2\pi}}$ $\quad (s = 2r)$

$s = \frac{1}{4}\left(L - 2\pi\frac{L}{8+2\pi}\right)$

$= \frac{1}{4}\frac{8L + 2\pi L - 2\pi L}{8+2\pi}$

$s = \frac{2L}{8+2\pi}$ $\quad \boxed{= \frac{L}{4+\pi}}$