CALCULUS I EXAM 6C

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Show all work on separate paper. Calculators and formula sheets are allowed. As always, you may keep this test.

1. Solve for the variables without using tables or calculator.

(3ea)

a) $y = e^{\ln 3.5}$ b) $\log x = \log 3 + \log 5 - 2\log 4$

c) $z = \pi \log_\pi\left(\frac{1}{\pi^4}\right)$ d) $W = e^{3\ln 2}$

2. Find the derivatives: (Be sure to simplify if possible!)

(3ea)

a) $\frac{d}{dx} \ln\left(\frac{x}{x+1}\right)$ b) $\frac{d}{dx} 3^{(x^2+4)}$ c) $\frac{d}{dx} \ln(1+e^{2x})$

d) $\frac{d}{dx} \ln[\ln(\sin x)]$ e) $\frac{d}{dx}(e^{-x}\cos 2x)$ f) $\frac{d}{dx} e^{(4+\ln x)}$

3. Find the integrals:

(5ea)

a) $\int \frac{x^2}{1+x^3}\,dx$ b) $\int \frac{1}{x(\ln x)^5}\,dx$ c) $\int x e^{x^2}\,dx$

d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ e) $\int_{\ln 3}^{\ln 5} \frac{e^x}{e^x+4}\,dx$

(4ea) 4a) Use logarithms to differentiate: $y = \dfrac{x^5\sqrt{x^4+8}}{(2x^3+4)\sqrt[3]{2x-1}}$
b) Find $y'(1)$.

(5ea) 5a) Show that $\log_a x = \frac{\ln x}{\ln a}$. [Hint: Begin $y = \log_a x$]
b) Show that if the growth of a population is proportional to the population P, then $P = P_0 e^{kt}$ where P_0 is the population at $t=0$.

(8) 6. Find the area between the curves $y = \ln x$, $y = e^x$, $x=0$, $y=0$, $x=4$.

(5ea) 7. A population of 10,000 takes 3 years to double. a) How long will it take to reach 100,000? b) What is the population after 20 years? [Assume growth rate proportional to the population.]

(3ea) 8. If $y = x^2 - 5x - 6$, a) determine intervals over which the function is 1-1.
b) determine the inverse over each interval $[x = g(y)]$
c) give the derivative of each inverse $\frac{dx}{dy}$ as a function in terms of x.

9. Find y' if $y = (\ln x)^x$

CALCULUS I EXAM 6C Solutions

NOT GUARANTEED (8-5-86) Dr RAPALJE

1. a) $y = e^{\ln 3.5}$

$y = 3.5$

b) $\log x = \log 3 + \log 5 - 2\log 4$

$\log x = \log \frac{15}{16}$

$x = \frac{15}{16}$

c) $z = \pi \log_\pi\left(\frac{1}{\pi^4}\right)$

$= \pi\left[\log_\pi 1 - \log_\pi \pi^4\right]$

$= \pi[0 - 4] = -4\pi$

d) $w = e^{3\ln 2} = e^{\ln 2^3} = 8$

2 a) $\frac{d}{dx}\ln\left(\frac{x}{x+1}\right)$

$= \frac{d}{dx}\left[\ln x - \ln(x+1)\right]$

$= \frac{1}{x} - \frac{1}{x+1}$

or $\frac{x+1-x}{x(x+1)} = \frac{1}{x(x+1)}$

b) $\frac{d}{dx}3^{(x^2+4)}$

$= 3^{(x^2+4)}2x\ln 3$

c) $\frac{d}{dx}\ln(1+e^{2x}) = \frac{1}{1+e^{2x}}\cdot 2e^{2x}$

d) $\frac{d}{dx}\ln\left[\ln(\sin x)\right]$

$= \frac{1}{\ln(\sin x)}\cdot\frac{1}{\sin x}\cdot\cos x$

or $\frac{\cot x}{\ln(\sin x)}$

e) $\frac{d}{dx}(e^{-x}\cos 2x) =$

$e^{-x}(-2\sin 2x) + \cos 2x(-e^{-x})$

$= -e^{-x}(2\sin 2x + \cos 2x)$

f) $\frac{d}{dx}e^{(4+\ln x)} = \frac{d}{dx}e^4\cdot e^{\ln x}$

$= \frac{d}{dx}(e^4 x) = e^4$

3a) $\int\frac{x^2}{1+x^3}dx$ let $u = 1+x^3$, $du = 3x^2dx$

$= \frac{1}{3}\int\frac{du}{u} = \frac{1}{3}\ln u + c$

$= \frac{1}{3}\ln|1+x^3| + c$

b) $\int\frac{1}{x(\ln x)^5}dx$ let $u = \ln x$, $du = \frac{1}{x}dx$

$= \int\frac{du}{u^5} = \int u^{-5}du$

$= \frac{u^{-4}}{-4} + c = -\frac{1}{4(\ln x)^4} + c$

c) $\int xe^{x^2}dx$ let $u = x^2$, $du = 2x\,dx$

$= \int e^u\frac{du}{2}$

$= \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + c$

d) $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ let $u = \sqrt{x}$, $du = \frac{1}{2}x^{-1/2}dx$

$2du = \frac{dx}{\sqrt{x}}$

$= \int e^u 2du$

$= 2e^u + c = 2e^{\sqrt{x}} + c$

e) $\int_{\ln 3}^{\ln 5}\frac{e^x}{e^x+4}dx$ let $u = e^x+4$, $du = e^x dx$

$= \int\frac{du}{u} = \ln u + c$

$= \ln(e^x+4)\Big|_{\ln 3}^{\ln 5}$

$= \ln(e^{\ln 5}+4) - \ln(e^{\ln 3}+4)$

$= \ln 9 - \ln 7$

$= \ln 9/7$

4. $\ln y = \ln\frac{x^5\sqrt{x^4+8}}{(2x^3+4)\sqrt[3]{2x-1}} = 5\ln x + \frac{1}{2}\ln(x^4+8) - \ln(2x^3+4) - \frac{1}{3}\ln(2x-1)$

$\frac{1}{y}y' = 5\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{x^4+8}\cdot 4x^3 - \frac{1}{2x^3+4}\cdot 6x^2 - \frac{1}{3}\frac{1}{2x-1}\cdot 2 \Rightarrow y' = \frac{x^5\sqrt{x^4+8}}{(2x^3+4)\sqrt[3]{2x-1}}\left[\frac{5}{x} + \frac{2x^3}{x^4+8} - \frac{6x^2}{2x^3+4} - \frac{2}{3(2x-1)}\right]$

$y'(1) = \frac{1\cdot 3}{6\cdot 1}\left[5 + \frac{2}{9} - \frac{6}{6} - \frac{2}{3}\right] = \frac{1}{2}\left(4 + \frac{2}{9} - \frac{2}{3}\right) = \frac{16}{9}$?

5a) let $y = \log_a x$

$a^y = x$ By def.

$\ln a^y = \ln x$ Take ln both sides.

$y\ln a = \ln x$ Property of logs.

$y = \frac{\ln x}{\ln a}$ solve for y.

b) $\frac{dP}{dt} = kP$ Given.

$\frac{dP}{P} = k\,dt$.

$\ln P = kt + c$

$P = e^{kt+c}$

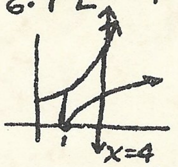
$P = e^{kt}\cdot e^c$

$P(0) = P_0$

$P_0 = e^0\cdot e^c$, so $e^c = P_0$

$P = P_0e^{kt}$

6.



$\int_0^1(e^x - 0)dx + \int_1^4(e^x - \ln x)dx$

$= e^x\Big|_0^1 + e^x - (x\ln x - x)\Big|_1^4$

$= (\cancel{e} - 1) + \left[e^4 - 4\ln 4 + 4 - \cancel{e} - 1\right]$

$= e^4 - 4\ln 4 + 2$

8. $y = (\ln x)^x$

$\ln y = \ln(\ln x)^x$

$\ln y = x\ln(\ln x)$

$\frac{1}{y}y' = 1\cdot\frac{1}{\ln x}\cdot\frac{1}{x} + \ln(\ln x)$

$y' = (\ln x)^x\left[\frac{1}{(\ln x)x} + \ln(\ln x)\right]$

7. $P = P_0e^{kt}$ $P_0 = 10{,}000$

$P = 2P_0$ at $t = 3$

$2P_0 = P_0e^{3k}$

$e^{3k} = 2$ or $e^k = \sqrt[3]{2}$

a) $100{,}000 = 10{,}000e^{kt}$

$10 = e^{kt}$

$10 = (e^k)^t = 2^{\frac{1}{3}t}$

$\ln 10 = \frac{1}{3}t\ln 2$

$t = \frac{3\ln 10}{\ln 2} = 9.97$ yrs.

b) $P = 10{,}000e^{20k}$

$= 10{,}000(e^k)^{20}$

$= 10{,}000(2^{1/3})^{20}$

$= 1015936$ Approx.

8. $y = x^2 - 5x - 6$

$\frac{dy}{dx} = 2x - 5 = 0$

$x = 5/2$

a) $(-\infty, 5/2]$ and $[\frac{5}{2}, \infty)$

b) $x^2 - 5x - 6 - y = 0$

$x = \frac{5 \pm \sqrt{25 - 4(-6-y)}}{2}$

$= \frac{5 \pm \sqrt{49+4y}}{2}$

b) $\frac{5 - \sqrt{49+4y}}{2}$ and $\frac{5 + \sqrt{49+4y}}{2}$

c) $\frac{dx}{dy} = \frac{-1}{\sqrt{49+4y}}$ and $\frac{+1}{\sqrt{49+4y}}$