CALCULUS II EXAM 4 Dr. Rapalje

Show all work on separate paper. Calculators (even TI-92) formula sheets are allowed. Justify all answers and name tests/theorems used. Explain use of TI-92!

In 1-2, write an expression for the general term of the sequence.

1a) $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \ldots$

b) $2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$

2a) $5, 10, 20, 40, \ldots$

b) $1, \frac{x}{2}, \frac{x^2}{6}, \frac{x^3}{24}, \frac{x^4}{120},$

19. A ball is dropped from a height of 100 feet. Each time it bounces to 2/3 of the previous height. Find the total distance traveled by the ball.

In 3-5, find the sum of the series if it exits:

3. $\frac{3}{25} - \frac{9}{125} + \frac{27}{625} - \ldots$

4. $1 + 0.9 + 0.81 + 0.729 + \ldots$

5. $\sum_{n=2}^{\infty} \frac{2^{n+3}}{3^n}$

6. $\sum_{n=0}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+4}\right)$

In 7-15, test for (absolute or conditional) convergence or divergence. NAME TESTS USED!

7. $\sum_{n=5}^{\infty} \frac{1}{n-4}$

8. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+3}$

9. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+4}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

11. $\sum_{n=2}^{\infty} \frac{e^n}{(\ln n)^n}$

12. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

13. $\sum_{n=0}^{\infty} \cos(n\pi)$

14. $\sum_{n=0}^{\infty} \frac{n^2 3^n}{n!}$

15. $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n$

16. Approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n 2^n}$ with an error of less than 0.001. Find n.

17. Approximate $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ with an error of less tha 0.001. Find n.

18. If $R_N \le \int_N^{\infty} f(x)\,dx$, $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, find N so $R_N \le 0.001$.

19. (See above)

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1a) 1 2 3 4 … n

$\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \ldots \frac{n}{n^2+1}$

b) 1 2 3 4 5 … n

$2 \quad -1 \quad \frac{1}{2} \quad -\frac{1}{4} \quad \frac{1}{8}$

$2^1 \quad -1 \quad \frac{1}{2^1} \quad -\frac{1}{2^2} \quad \frac{1}{2^3} \ldots \frac{(-1)^{n+1}}{2^{n-2}}$

2a) 1 2 3 4 n

$5, 10, 20, 40 \ldots$

$5\cdot1 \quad 5\cdot2 \quad 5\cdot4 \quad 5\cdot8 \quad 5\cdot2^{n-1}$ or $\frac{5}{2}\cdot 2^n$

b) 1 2 3 4 5 n

$1, \frac{x}{2}, \frac{x^2}{6}, \frac{x^3}{24}, \frac{x^4}{120} \quad \frac{x^{n-1}}{n!}$

3. $\frac{3}{25} - \frac{9}{125} + \frac{27}{625} - \ldots$

$\frac{3}{5^2} - \frac{3^2}{5^3} + \frac{3^3}{5^4} - \ldots = \sum_{n=1}^{\infty} \frac{3^n(-1)^{n+1}}{5^{n+1}}$

Geometric:

$a = \frac{3}{25} \quad r = -3/5 \quad S = \frac{a}{1-r} = \frac{3/25}{1+3/5} = \frac{3}{\cancel{25}_5}\cdot\frac{\cancel{5}}{8} = \frac{3}{40}$

4. $1 + 0.9 + 0.81 + 0.729 = \sum_{n=0}^{\infty} (.9)^n$

Geometric: $a=1$ $r=.9$

$S = \frac{1}{1-.9} = \frac{1}{.1} = 10$

5. $\sum_{n=2}^{\infty} \frac{2^{n+3}}{3^n} = \frac{32}{9} + \frac{64}{27} + \ldots$

Geometric $a = \frac{32}{9}$ $r = \frac{2}{3}$

$S = \frac{32/9}{1-2/3} = \frac{32}{9}\cdot\frac{3}{1} = \frac{32}{3}$

6. $\sum_{n=0}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n+4}\right)$ Telescoping.

$= \frac{1}{2} - \cancel{\frac{1}{4}} + \frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{5}} - \frac{1}{7} + \cancel{\frac{1}{6}} - \frac{1}{8} \ldots$

$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

7. $\sum_{n=5}^{\infty} \frac{1}{n-4}$ Integral test

$\int_5^{\infty} \frac{1}{x-4}dx = \ln(x-4)\Big|_5^{\infty} = \infty$

DIVERGES

Also P-series $p=1$

Limit comparison Test.

8. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+3}$

DIVERGENCE THEOREM

(N^{th} TERM!)

$\lim_{n\to\infty} a_n = 1 \neq 0.$

9. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+4}$

ABSOLUTELY CONVERGENT

$\int_0^{\infty} \frac{dx}{x^2+4} = \frac{1}{2}\arctan\frac{x}{2}\Big|_0^{\infty} = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$

INTEGRAL TEST OR COMPARISON TO P SERIES. $P=2$

$\sum_{n=1}^{\infty}\frac{1}{n^2+4} < \sum_{n=1}^{\infty}\frac{1}{n^2}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

CONVERGES by ALT SERIES THM

$\int_1^{\infty} \frac{x\,dx}{x^2+1} = \frac{1}{2}\int\frac{du}{u}$

$= \frac{1}{2}\ln(x^2+1)\Big|_1^{\infty}$

$= \infty$ DIVERGES BY INTEGRAL TEST.

$\therefore$ CONDITIONALLY CONVERGENT

11. $\sum_{n=2}^{\infty} \frac{e^n}{(\ln n)^n}$

ROOT TEST:

$\lim_{n\to\infty} \sqrt[n]{a^n}$

$= \lim_{n\to\infty} \sqrt[n]{\frac{e^n}{(\ln n)^n}}$

$= \lim_{n\to\infty} \frac{e}{\ln n} = 0 < 1$

CONVERGES

12. $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

INTEGRAL TEST: $\int_2^{\infty}\frac{dx}{x\ln x}$ Let $u = \ln x$ $du = \frac{1}{x}dx$

$= \int\frac{du}{u} = \ln(\ln x)\Big|_2^{\infty}$

$= \infty$

DIVERGES

13. $\sum_{n=0}^{\infty} \cos(n\pi)$

$= 1 - 1 + 1 - 1 \ldots$

DIVERGES DIV. THM or N^{th} TERM.

$\lim_{n\to\infty} a_n \neq 0.$

14. $\sum_{n=0}^{\infty} \frac{n^2 3^n}{n!}$

RATIO TEST

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

$= \lim_{n\to\infty} \frac{(n+1)^2 \cancel{3^{n+1}}}{(n+1)!}\cdot\frac{n!}{n^2\cancel{3^n}}$

$= \lim_{n\to\infty} \frac{3(n+1)^2\cdot\cancel{n!}}{(n+1)\cdot\cancel{n!}\,n^2}$

$= \lim_{n\to\infty} \frac{3(n+1)}{n^2} = \frac{\infty}{\infty}.$

$= \lim_{n\to\infty} \frac{3}{2n}$ (L'Hopital)

$= 0$ CONVERGES

15. $\sum_{n=1}^{\infty}\left(1+\frac{3}{n}\right)^n$

DIVERGENCE THM.

$\lim_{n\to\infty} a_n \neq 0$

16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$

Alt. series

$|R_N| < |a_{n+1}| = .001$

$a_{n+1} = \frac{1}{(n+1)2^{n+1}}$

$n=7$ TRIAL & ERROR

$\frac{1}{7\cdot128} = \frac{1}{896} \leq .001$

$= \frac{1}{2} - \frac{1}{2\cdot4} - \frac{1}{3\cdot8} + \frac{1}{4\cdot16} - \frac{1}{5\cdot32} - \frac{1}{6\cdot64} + \frac{1}{7\cdot128}$

$= .405804 \; (n=7)$ $\quad .406$

$= .405465 \; (n=100)$ $\quad n=6$

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17. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ $\quad |a_{n+1}| = \frac{1}{2(n+1)+1}$

$$= \frac{1}{2n+3} < .001$$

$$2n+3 > 1000$$

$$2n > 997$$

$$n > 498.5$$

$$n = 499$$

$\Sigma((-1)\wedge n/(2n+1), n, 0, 499)$ ◆ ENTER $= .784898$

18. $R_N \leq \int_N^{\infty} \frac{1}{x^2+1}\,dx = \arctan x \Big|_N^{\infty} \leq .001$

$$\frac{\pi}{2} - \arctan N \leq .001$$

$$-\arctan N \leq .001 - \frac{\pi}{2}$$

$$\arctan N \geq \frac{\pi}{2} - .001$$

$\Sigma(1/(x\wedge 2+1), x, 1, 1000)$ $\quad N \geq \tan\left(\frac{\pi}{2} - .001\right)$

$= 1.07567$ $\quad N \geq 1000$ (NOTE: Huge roundoff error in Solution Manual for #48 p. 574. They have $N \geq 1004$.)

CONVERGES VERY SLOWLY!!

19.

$$100 + 2\left[\frac{2}{3}\cdot 100 + \frac{2}{3}\cdot\frac{2}{3}\cdot 100 + \ldots\right]$$

$$= 100 + 2\left[\frac{200}{3}\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots\right)\right]$$ Geometric $a = 1,\ r = \frac{2}{3}$

$$= 100 + \frac{400}{3}\left(\frac{1}{1-\frac{2}{3}}\right) \text{ or } 100 + \frac{400}{3}\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n$$

$$= 100 + \frac{400}{3}\cdot 3 \qquad 100 + \frac{400}{3}\cdot 3$$

$$= 100 + 400 = 500 \text{ ft}$$