CALCULUS II EXAM 2 A Dr. RAPALJE--SCC at HUNT CLUB

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers. Where calculators are used, describe window, procedures, etc. Gold Sheet, Supplement to Gold Sheet, Trig Sheet, Chapter 6 Fomula Sheet are allowed.

1. **Give the set-up to find the area of the region above the X-axis, bounded by Y=X-2 and X=Y^2 a) using vertical slices b) using horizontal slices; c) calculate the easiest way.**

2. **Find the area bounded by** $y = \frac{4 \ln x}{x}$**, y=0, and x=5. Give exact value. Check with calculator.**

3. **Find the volume of the solid obtained by rotating the region bounded by y=x^3, y=0, and x=2 about the x axis. Give a set-up for a) disk method; b) shell method; c) solve easiest way.**

4. **Find the volume of the solid obtained by rotating the region bounded by y=x^3, y=0, and x=2 about the y axis. Give a set-up for a) disk method; b) shell method; c) solve easiest way.**

5. **Find the arclength of the curve** $y = \ln(x + \sqrt{x^2 - 1})$ **from x=3 to x=5, given that** $y' = \frac{1}{\sqrt{x^2 - 1}}$**. a) Use the calculator to find the decimal value; b) Integrate to obtain exact value.**

6. **Find the surface area obtained by rotating the curve** $x = \frac{y^2}{4} - \frac{1}{2} \ln y$ **from y=1 to y=2 about the x-axis.**

7. **Find the center of mass of the region shown in the graph:**

8. Find the center of mass of the region: $y=4-x^2$, $y=0$, $x=0$. (Use calculator to evaluate the integrals.)

9. Use the Theorem of Pappus to find the volume of the torus formed by rotating the circle $(x - 5)^2 + (y - 4)^2 = 4$
A) about the x axis;   B) about the y axis.

10. Use the disk or shell method and the equation $x^2 + y^2 = r^2$ to derive the formula for the volume of a sphere. Explain your method (disk, shell, axis of rotation, etc.).

CALCULUS II EXAM 2 Solutions

6

1.a) Vertical slices: $y = \pm\sqrt{x}$

$$A = \int_0^1 \left[\sqrt{x} - (-\sqrt{x})\right] dx + \int_1^4 \left[\sqrt{x} - (x-2)\right] dx$$

b) Horizontal slices: $x = y+2$

$$A = \int_{-1}^{2} \left[(y+2) - y^2\right] dy$$

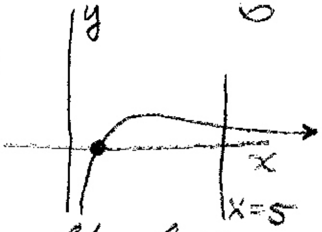
c) $= \left.\frac{y^2}{2} + 2y - \frac{y^3}{3}\right|_{-1}^{2}$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$

$= 8 - \frac{1}{2} - \frac{9}{3} = 9/2$

2. $A = \int_1^5 \frac{4 \ln x}{x} dx$



$= \int 4u\, du$

Let $u = \ln x$
$du = \frac{1}{x} dx$

$= 2u^2 + c$

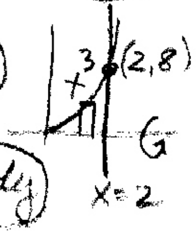
$= 2(\ln x)^2 \Big|_1^5$

$= 2(\ln 5)^2 = 5.1806$

$\boxed{2^{nd}}$, $\boxed{\text{CALC}}$, $\boxed{\text{F5}}$, $\boxed{4(\ln x)/x}$ $\boxed{, x, 1, 5)}$ ENTER

3.
a) $V = \pi \int_0^2 (x^3)^2 dx$

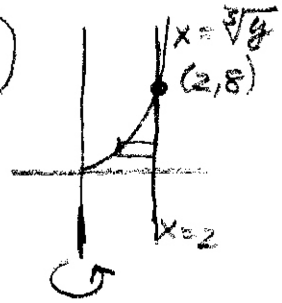
b) $V = 2\pi \int_0^8 \left(2 - \sqrt[3]{y}\right) y\, dy$



c) $V = \pi \left.\frac{x^7}{7}\right|_0^2 = \frac{128\pi}{7} \approx 57.4463$

4.
a) $V = \pi \int_0^8 \left[2^2 - \left(\sqrt[3]{y}\right)^2\right] dy$

b) $V = 2\pi \int_0^2 x(x^3)\, dx$



c) $= 2\pi \left.\frac{x^5}{5}\right|_0^2$

$= \frac{2\pi}{5} \cdot 32 = \frac{64\pi}{5} \approx 40.2124$

5. $y = \ln\left(x + \sqrt{x^2-1}\right)$

$y' = \frac{1}{\sqrt{x^2-1}}$

$s = \int_3^5 \sqrt{1 + \frac{1}{x^2-1}}\, dx$

$= \int_3^5 \sqrt{\frac{x^2-1+1}{x^2-1}}\, dx$

$= \int_3^5 \frac{x}{\sqrt{x^2-1}}\, dx$ Let $u = \sqrt{x^2-1}$

$u^2 = x^2-1$

$2u\,du = 2x\,dx$

$u\,du = x\,dx$

$= \int \frac{u\,du}{u}$

$= \int du = u = \sqrt{x^2-1}\Big|_3^5$

$= \sqrt{24} - \sqrt{8}$

$= 2\sqrt{6} - 2\sqrt{2} \approx 2.0706$

2^{nd}, CALC, F5, $x/(x^2-1)^{1/2}$, x, 3, 5) Enter

6. $A = 2\pi \int y\, ds$ $\quad x = \frac{y^2}{4} - \frac{1}{2}\ln y$

$= 2\pi \int_1^2 y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$ $\quad \frac{dx}{dy} = \frac{y \cdot y}{2 \cdot y} - \frac{1}{2y}$

$= 2\pi \int_1^2 y\sqrt{1 + \left(\frac{y^2-1}{2y}\right)^2}\, dy$ $\quad = \frac{y^2-1}{2y}$

$= 2\pi \int_1^2 y\sqrt{1 + \frac{y^4 - 2y^2 + 1}{4y^2}}\, dy$

$= 2\pi \int_1^2 y\sqrt{\frac{4y^2 + y^4 - 2y^2 + 1}{4y^2}}\, dy$

$= 2\pi \int_1^2 \cancel{y}\, \frac{\sqrt{y^4 + 2y^2 + 1}}{2\cancel{y}}\, dy = \pi \int_1^2 \sqrt{(y^2+1)^2}\, dy$

$= \pi \int_1^2 (y^2+1)\, dy = \pi\left[\frac{y^3}{3} + y\right]_1^2$

$= \pi\left[\frac{8}{3} + 2 - \left(\frac{1}{3} + 1\right)\right]$

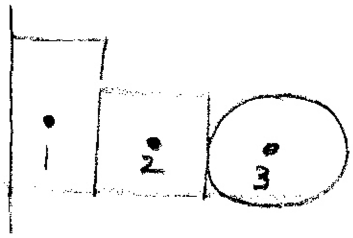
$= \pi\left[\frac{7}{3} + 1\right] = \frac{10\pi}{3} \approx 10.47197$

or fnInt $(2\pi x \sqrt{\ }(1 + ((x^2-1)/(2x))^2), x, 1, 2)$

– OR –

2^{nd}, CALC, MORE, F3 (arc), $\ln(x + (x^2-1)^\wedge.5)$, x, 3, 5) Enter.

7.



A_1 = Rectangle $= 2\times 6 = 12$
cm $(1,3)$

A_2 = Square $= 4\times 4 = 16$
cm $=(4,2)$

A_3 = Circle $= \pi r^2 = 4\pi$
cm $=(8,2)$

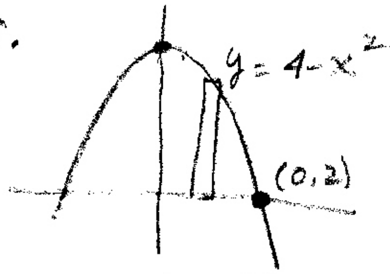
$$\overline{x} = \frac{12\cdot 1 + 16\cdot 4 + 4\pi\cdot 8}{12+16+4\pi}$$

$$= \frac{76+32\pi}{28+4\pi} = \frac{19+8\pi}{7+\pi} \approx 4.3577$$

$$\overline{y} = \frac{12\cdot 3 + 16\cdot 2 + 4\pi\cdot 2}{28+4\pi}$$

$$= \frac{68+8\pi}{28+4\pi} = \frac{17+2\pi}{7+\pi} \approx 2.2958$$

8.



$$\overline{y} = \frac{\int_0^2 \frac{(4-x^2)^2}{2}\,dx}{16/3}$$

$$= \frac{8.53333}{16/3} \text{ (FRAC)}$$

$$= \frac{128/15}{16/3} = 8/5$$

1.6

$$\overline{x} = \frac{\int_0^2 x(4-x^2)\,dx}{\int_0^2 (4-x^2)\,dx}$$

$$= \frac{\int_0^2 4x - x^3\,dx}{\int_0^2 (4-x^2)\,dx}$$

$$= \frac{2x^2 - \frac{x^4}{4}\Big|_0^2}{4x - \frac{x^3}{3}\Big|_0^2}$$

$$= \frac{8-4}{8-8/3} = \frac{4}{16/3} = \frac{3}{4}$$

–or–

$$\overline{y} = \frac{\int_0^2 \frac{4-x^2}{2}(4-x^2)\,dx}{\int_0^2 (4-x^2)\,dx}$$

$$= \frac{\frac{1}{2}\int_0^2 (16-8x^2+x^4)\,dx}{16/3}$$

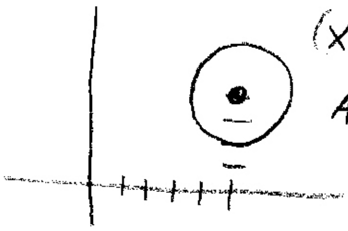
$$= \frac{\frac{1}{2}\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2}{16/3}$$

$$= \frac{\frac{1}{2}\left[32 - \frac{64}{3} + \frac{32}{5}\right]}{16/3} = \frac{16 + \frac{16}{5} - \frac{32}{3}}{16/3}$$

$$= \frac{\frac{240+48-160}{15}}{16/3} = \frac{128}{15}\cdot\frac{3}{16}$$

$$= 8/5$$

9.



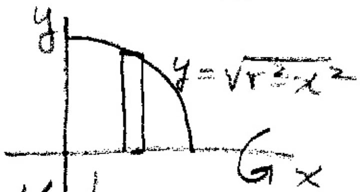
$(x-5)^2 + (y-4)^2 = 4 \quad C(5,4)$

$A = \pi R^2 \quad V = \text{Area} \times 2\pi r$
$= 4\pi$

a) x axis: radius $= 4$
$V = \text{Area} \times 2\pi r$
$= 4\pi\cdot 2\pi\cdot 4 = 32\pi^2$

b) y axis: radius $= 5$
$V = 4\pi\cdot 2\pi\cdot 5 = 40\pi^2$

10. $x^2+y^2=r^2$
$y = \sqrt{r^2-x^2}$



$V_{\text{Hemisphere}}$ x axis Disk method

$$= \pi\int_0^r \left(\sqrt{r^2-x^2}\right)^2 dx$$

$$= \pi\int_0^r (r^2-x^2)\,dx$$

$$= \pi\left[r^2x - \frac{x^3}{3}\right]_0^r = \pi\left[r^3 - \frac{r^3}{3}\right] = \frac{2\pi r^3}{3}$$

$$V_{\text{sphere}} = 2\cdot\frac{2\pi r^3}{3} = \frac{4\pi r^3}{3}$$