CALCULUS II EXAM 6 A Dr. RAPALJE--SCC at HUNT CLUB

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers. Where calculators are used, especially TI-92s, describe your procedures, etc. Approved formula sheets are allowed.

1. Name the conic, find its center, vertices, and foci. Express in standard form and sketch the graph.
$9x^2 - y^2 - 36x - 6y + 18 = 0$.

2. Name the conic, find the angle of the rotated system, sketch the graph, and find the coordinates of the focus (foci) in $(\overline{x}, \overline{y})$ and $(x, y)$.
$8x^2 - 4xy + 5y^2 = 36$

3a) Sketch the curve (give range values) for
$x = t^2 + 3t \quad y = t + 1$

b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

c) Find an xy equation.

4a) Sketch the curve $r = 2\cos 2\theta$ (give range values)

b) Find all tangents at the pole for $0 \le \theta \le 2\pi$.

c) Write an integral to find the area of one petal.

d) Find the area by calculator and also by integration technique.

5a) Sketch the curve $r = 2 - 4\sin\theta$ (give range values)

b) Set up an integral (and simplify) to find the arc length of the inner loop.

6. Given $r = 4 - 4\sin\theta$

a) Find the area enclosed by the arc

b) Find the length of the arc.

Explain what you did.

CALCULUS II EXAM 5A Solutions Dr. Rapalje

1. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n\,5^n}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{(n+1)\,5^{n+1}} \cdot \frac{n\,5^n}{(x-3)^n} \right| < 1$

$= \lim_{n\to\infty} \left| \frac{n}{n+1} \frac{(x-3)}{5} \right| < 1$

$|x-3| < 5$

$-5 < x-3 < 5$

$-2 < x < 8$

Endpts:

$x=-2$

$\sum_{n=1}^{\infty} \frac{(-1)^n(-5)^n}{n\,5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ Diverges

$x=8$

$\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{n\,5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Converges

$-2 < x \le 8$

2. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{x^{2n+2}}{(n+1)!} \cdot \frac{n!}{x^{2n}} \right| < 1$

$= \lim_{n\to\infty} \left| \frac{x^2}{n+1} \right| < 1$

All real x

$-\infty < x < \infty$

3. $f(x) = \frac{-\frac{1}{5}\cdot 1}{-\frac{1}{5}(-5+2x)} = \frac{-\frac{1}{5}}{1-\frac{2x}{5}}$

Geometric with $a = -\frac{1}{5}$, $r = \frac{2x}{5}$

$= \sum_{n=0}^{\infty} -\frac{1}{5}\left(\frac{2x}{5}\right)^n = \sum_{n=0}^{\infty} \frac{-2^n x^n}{5^{n+1}}$

4. $f(x) = \frac{4}{4+x^2} = \frac{1}{1+\frac{x^2}{4}}$

$\sum_{n=0}^{\infty} 1\cdot\left(\frac{-x^2}{4}\right)^n = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^{2n}$

5. $f(x) = x^{-1}$ $\quad f(1) = 1$

$f'(x) = -x^{-2}$ $\quad f'(1) = -1$

$f''(x) = 2x^{-3}$ $\quad f''(1) = 2$

$f'''(x) = -3\cdot 2\cdot x^{-4}$ $\quad f'''(1) = -3!$

$f^{n}(x) = \frac{(-1)^n n!}{x^{n+1}}$ $\quad f^{n}(1) = (-1)^n n!$

$f(x) = 1 - 1(x-1) + \frac{\cancel{2}}{\cancel{2!}}(x-1)^2 - \frac{\cancel{3!}}{\cancel{3!}}(x-1)^3 + \dots$

$= \sum_{n=0}^{\infty} (-1)^n (x-1)^n$

6. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$

$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

7. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

$\cos t^2 = 1 - \frac{t^4}{2!} + \frac{t^8}{4!} - \frac{t^{12}}{6!} + \dots = \sum_{n=0}^{\infty} \frac{t^{4n}}{(2n)!}$

$\int \cos t^2\,dt = t - \frac{t^5}{5\cdot 2!} + \frac{t^9}{9\cdot 4!} - \frac{t^{13}}{13\cdot 6!} + \dots = \sum_{n=0}^{\infty} \frac{t^{4n+1}(-1)^n}{(4n+1)(2n)!} + C$

8. $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}$

$\frac{\arctan t}{t} = 1 - \frac{t^2}{3} + \frac{t^4}{5} - \frac{t^6}{7} \dots$

$= \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{2n+1}$

$\int \frac{\arctan t}{t}\,dt = t - \frac{t^3}{3\cdot 3} + \frac{t^5}{5\cdot 5} - \frac{t^7}{7^2}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)^2} + C$