CALCULUS II EXAM 1 B Term I 1983-84 Dr. RAPALJE

Show all work on separate paper. (One problem will be omitted)

1. Use the 3 step rule to show that if $f(x) = \cos x$, then $f'(x) = -\sin x$.

2. Use $\csc x = \frac{1}{\sin x}$ to prove that if $f(x) = \csc x$ then $f'(x) = -\csc x \cot x$

3. a) Find $\lim_{\theta \to 0} (\sin 2\theta \cot \theta)$ b) Find $\lim_{\theta \to 0} \frac{\theta^2}{1-\cos\theta}$

4. If $f(x) = (\cos \sqrt{x})^3$, find $f'(x)$.

5. If $f(x) = \frac{\sin 2x}{1+\cos 2x}$, find $g'(x)$.

6. If $y = \operatorname{arccsc} x$, show that $\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$
Hint: Let $x = \csc y$ and differentiate implicitly.

7. If $y = \arctan \frac{x}{\sqrt{1-x^2}}$, find y'. Simplify.

8. $\int \frac{\sin 2x}{\cos^7 2x}\, dx$

9. $\int \cos^3 x \, dx$

10. $\int \sec^5 3x \tan 3x \, dx$

11. $\int \frac{dx}{16+9x^2}$

CALCULUS II EXAM 1_B Solutions

Dr. RAPALJE

1. $f(x) = \cos x$

$$f(x+h) = \cos(x+h) = \cos x \cosh - \sin x \sinh$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \cos x \underbrace{\frac{(\cos h - 1)}{h}}_{=0} - \sin x \underbrace{\frac{\sin h}{h}}_{=1}$$

$$\lim_{h \to 0} = \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

2. $f(x) = \csc x = \frac{1}{\sin x}$

$$f'(x) = \frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x}$$

(Quotient rule)

$$= -\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= -\csc x \cot x$$

3a) $\lim_{\theta \to 0} \sin 2\theta \cot\theta$

$$= \lim_{\theta \to 0} 2 \cancel{\sin\theta} \cos\theta \cdot \frac{\cos\theta}{\cancel{\sin\theta}}$$

$$= \lim_{\theta \to 0} 2\cos^2\theta = 2$$

b) $\lim \frac{\theta^2}{(1-\cos\theta)} \frac{(1+\cos\theta)}{(1+\cos\theta)}$

$$= \lim_{\theta \to 0} \frac{\theta^2}{\sin^2\theta}(1+\cos\theta)$$

$$= 1 \cdot (1+1) = 2$$

4. $f(x) = \cos^3\sqrt{x}$

$$f'(x) = 3\cos^2\sqrt{x}(-\sin\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$$

$$= -\frac{3\cos^2\sqrt{x}\,\sin\sqrt{x}}{2\sqrt{x}}$$

5. $g(x) = \frac{\sin 2x}{1+\cos 2x}$

Alt. solution: $g(x) = \frac{2\sin x \cos x}{1+2\cos^2 x - 1} = \tan x$

$$g'(x) = \sec^2 x$$

$$g'(x) = \frac{(1+\cos 2x)2\cos 2x - \sin 2x(-\sin 2x) \cdot 2}{(1+\cos 2x)^2}$$

$$= \frac{2\cos 2x + 2\cos^2 2x + 2\sin^2 2x}{(1+\cos 2x)^2}$$

$$= \frac{2\cos 2x + 2(\cos^2 2x + \sin^2 2x)}{(1+\cos 2x)^2}$$

$$= \frac{2(\cos 2x + 1)}{(1+\cos 2x)^2} = \frac{2}{1+\cos 2x}$$

6. $x = \csc y$

$$1 = -\csc y \cot y \, y'$$

$$y' = \frac{1}{-\csc y \cot y}$$

$$\cot^2 y = \csc^2 y - 1 = x^2 - 1$$

$$\cot y = \sqrt{x^2-1}$$

$$= \frac{1}{-x\sqrt{x^2-1}}$$

7. $y = \arctan\frac{x}{\sqrt{1-x^2}}$

$$y' = \frac{1}{1+\frac{x^2}{1-x^2}} \cdot \frac{\sqrt{1-x^2} \cdot 1 - x \cdot \frac{-x}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{1}{\frac{1-x^2+x^2}{1-x^2}} \cdot \frac{1-x^2+x^2}{(1-x^2)^{3/2}}$$

$$= \frac{1-x^2}{1} \cdot \frac{1}{(1-x^2)^{3/2}} = \frac{1}{\sqrt{1-x^2}}$$

8. $\int \frac{\sin 2x}{\cos^7 2x}\,dx$ $\quad u = \cos 2x$, $du = -\sin 2x \, 2\,dx$

$$= \int u^{-7}\left(-\frac{du}{2}\right)$$

$$= \frac{-u^{-6}}{-12} + C = \frac{1}{12u^6} + C$$

$$= \frac{1}{12\cos^6 2x} + C \text{ or } \frac{1}{12}\sec^6 2x + C$$

9. $\int \cos^3 x \, dx = \int (1-\sin^2 x)\cos x \, dx$ $\quad$ Let $u = \sin x$, $du = \cos x \, dx$

$$= \int 1 - u^2 \, du$$

$$= u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$$

10. $\int \sec^5 3x \tan 3x \, dx$ $\quad$ Let $u = \sec 3x$, $du = 3\sec 3x \tan 3x \, dx$

$$= \int \sec^4 3x \sec 3x \tan 3x \, dx$$

$$= \int u^4 \frac{du}{3} = \frac{u^5}{15} + C = \frac{1}{15}\sec^5 3x + C$$

11. $\int \frac{dx}{16+9x^2}$ $\quad$ Let $a = 4$, $u = 3x$, $du = 3\,dx$, $\frac{du}{3} = dx$

$$\frac{1}{3}\int \frac{du}{16+u^2} = \frac{1}{12}\arctan\frac{3x}{4} + C$$