CALCULUS II EXAM 2B NAME Dr. RAPALJE 

Show all work on separate paper.
Calculators, formula sheets, tables from book are allowed.

1. If $\ln 2 = .7$, $\ln 3 = 1.1$, and $\ln 10 = 2.3$, use the laws of logarithms to find
   a) $\ln 24$ b) $\ln \sqrt[3]{15}$

2. Find the derivatives:
   a) $f(x) = \ln(\sec x + \tan x)$
   b) $f(x) = x^3 e^{x^2}$
   c) $f(x) = x^{\ln x}$ $\Longrightarrow$ d) $f(x) = e^{-\ln(\sin x)}$

3. Find the integral:
   a) $\int e^{\tan x} \sec^2 x \, dx$
   b) $\int \frac{\sin x}{1+\cos x} dx$
   c) $\int \frac{dx}{x^2 \cos(1/x)}$ $\Longrightarrow$ d) $\int \frac{2x+4}{x^2+1} dx$

4. Prove that if $y = e^x$, then $y' = e^x$.
   Give reasons for each step.

5. Find $f'(x)$, $f''(x)$, relative max, min, points of inflection, and graph if $f(x) = x^2 e^{-x}$.

1(a) $\ln 24 = \ln 8\cdot 3$
$= 3\ln 2 + \ln 3$
$= 3(.7) + 1.1$
$= 3.2$

b) $\ln \sqrt[3]{15} = \frac{1}{3}\ln \frac{30}{2}$
$= \frac{1}{3}(\ln 10 + \ln 3 - \ln 2)$
$= \frac{1}{3}(2.3 + 1.1 - .7)$
$= .9$

2a) $f(x) = \ln(\sec x + \tan x)$
$f'(x) = \frac{1}{\sec x + \tan x}\cdot \sec x \tan x + \sec^2 x$
$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \sec x$

b) $f(x) = x^3 e^{x^2}$
$f'(x) = x^3\cdot e^{x^2}\cdot 2x + 3x^2 e^{x^2}$
$= x^2(2x^2+3)e^{x^2}$

2c) $f(x) = x^{\ln x}$
$\ln f(x) = \ln x^{\ln x}$
$= (\ln x)\ln x$
$\ln f(x) = (\ln x)^2$
$\frac{1}{f(x)}\cdot f'(x) = 2\ln x\cdot\frac{1}{x}$
$f'(x) = \frac{x^{\ln x}\cdot 2\ln x}{x}$

d) $f(x) = e^{-\ln(\sin x)}$
$= e^{\ln(\sin x)^{-1}}$
$= \csc x$
$f'(x) = -\csc x \cot x$

3a) $\int e^{\tan x}\sec^2 x\,dx$  $u = \tan x$
$du = \sec^2 x\,dx$
$\int e^u\,du = e^{\tan x} + C$

b) $\int \frac{\sin x}{1+\cos x}dx$  $u = 1+\cos x$
$du = -\sin x\,dx$
$\int -\frac{du}{u} = -\ln|1+\cos x| + C$

3c) $\int \frac{dx}{x^2\cos\frac{1}{x}}$  Let $u = x^{-1}$
$du = -x^{-2}dx$
$= \int -\frac{du}{\cos u}$  $\frac{-du}{1} = \frac{dx}{x^2}$
$= -\int \sec u\,du = -\ln\left|\sec\frac{1}{x} + \tan\frac{1}{x}\right| + C$

4. $y = e^x$
$\ln y = \ln e^x$  Take ln both sides.
$\ln y = x$  Since $\ln e^x = x$.
$\frac{1}{y}y' = 1$  Implicit differentiation.
$y' = y = e^x$  substitution.

3d) $\int \frac{2x+4}{x^2+1}dx = \int\frac{2x\,dx}{x^2+1} + \int\frac{4}{x^2+1}dx$
$= \int\frac{2x\,dx}{x^2+1}$  Let $u = x^2+1$
$du = 2x\,dx$
$+ 4\int\frac{dx}{x^2+1}$ (see arctan formula)
$= \ln|x^2+1| + 4\arctan x + C$

5. $f(x) = x^2 e^{-x}$
$f'(x) = x(2-x)e^{-x} = 0$ at $x = 0, 2$
$f''(x) = (x^2-4x+2)e^{-x} = 0$ at $x = 2\pm\sqrt{2}$
$= 3.4, .6$

| x | -1 | 0 | .6 | 2 | 3.4 | 4 |
|---|---|---|---|---|---|---|
| f | //// | 0 | .2 | .5 | .4 | //// |
| f' | − | 0 | + | 0 | − | − |
| f'' | + | + | 0 | − | 0 | + |

Rel max at $x = 2$
Rel min at $x = 0$
Pt of infl at $x = 2\pm\sqrt{2}$