CALCULUS II EXAM 4B (Polar Coordinates) NAME Dr. RAPALJE

PART I: Graph and name the graph. If a conic, give both focus points.

1. $r = -2(1+\cos\theta)$

2. $r = \dfrac{10}{2+3\sin\theta}$

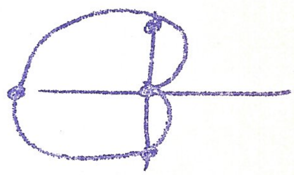
3. $r = \dfrac{6}{2\cos\theta - 3\sin\theta}$ Give (x,y) equation.

4. $r = -6\cos\theta$ Give (x,y) equation.

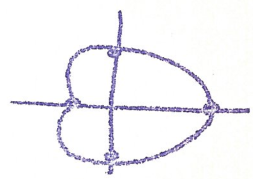
PART II:

5. Find $\frac{ds}{d\theta}$ and $\cot\psi$ for

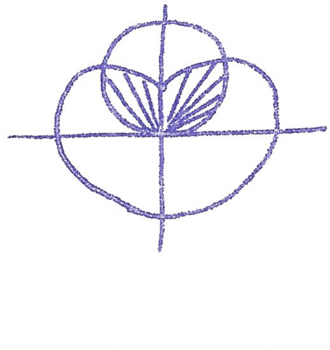
6. Find the arclength of $r = a(1-\cos\theta)$



7. Find the area enclosed by $r = 2 + \cos\theta$



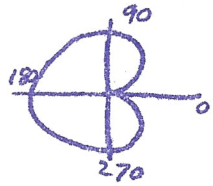
8. Find the area shaded:
   $r = 3\sin\theta$, $r = 2 - \sin\theta$



CALCULUS II Polar Coord. Exam 4B Solutions Dr. RAPALJE

1. $r = -2(1+\cos\theta)$ cardioid

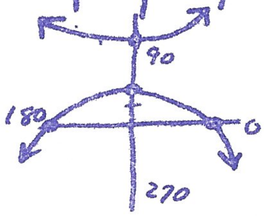
θ	0	30°	60°	90°	180°	Symmetry.
r	−4	−3.7	−3	−2	0	



2. $r = \dfrac{10}{2+3\sin\theta} = \dfrac{5}{1+\frac{3}{2}\sin\theta}$

$e = 3/2$ Hyperbola Focus $(0,0)$

θ	0	90°	180	270
r	5	2	5	−10



Other focus at $10+2=12$

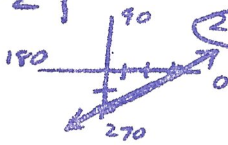
$F(12, \pi/2)$

3. $r = \dfrac{6}{2\cos\theta - 3\sin\theta}$ St. line

θ	0	$\pi/2$
r	3	−2

$2r\cos\theta - 3r\sin\theta = 6$

$2x - 3y = 6$



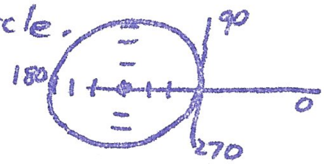
4. $r = -6\cos\theta$ Circle.

$r^2 = -6r\cos\theta$

$x^2 + y^2 = -6x$

$x^2 + 6x + 9 + y^2 = 9$

$C(-3,0)$ $r = 3$ in (x,y) or (r,θ)



5. $r(1+\cos\theta) = 3$

$r = 3(1+\cos\theta)^{-1}$

$\dfrac{dr}{d\theta} = -3(1+\cos\theta)^{-2}\cdot(-\sin\theta)$

$= \dfrac{3\sin\theta}{(1+\cos\theta)^2}$

$\cot\psi = \dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{1+\cos\theta}{3}\cdot\dfrac{3\sin\theta}{(1+\cos\theta)^2}$

$= \dfrac{\sin\theta}{(1+\cos\theta)}$

$\dfrac{ds}{d\theta} = \sqrt{\dfrac{9}{(1+\cos\theta)^2} + \dfrac{9\sin^2\theta}{(1+\cos\theta)^4}}$

$= \sqrt{\dfrac{9(1+2\cos\theta+\cos^2\theta)+9\sin^2\theta}{(1+\cos\theta)^4}}$

$= \dfrac{\sqrt{18+18\cos\theta}}{(1+\cos\theta)^2} = \dfrac{3\sqrt{2}}{(1+\cos\theta)^{3/2}}$

6. $r = a(1-\cos\theta) = 0$ at $\cos\theta = 1$

$\theta = 0, 2\pi$

$\dfrac{dr}{d\theta} = a\sin\theta$

$\int_0^{2\pi}\sqrt{r^2+(r')^2}\,d\theta$

$= \int_0^{2\pi}\sqrt{a^2(1-2\cos\theta+\cos^2\theta)+a^2\sin^2\theta}\,d\theta$

$= \int_0^{2\pi}\sqrt{a^2(2-2\cos\theta)}\,d\theta$

$\sin\frac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}}$

$\sqrt{2}\sin\frac{\theta}{2} = \sqrt{1-\cos\theta}$

$= \int_0^{2\pi} a\sqrt{2}\sqrt{1-\cos\theta}\,d\theta$

$= a\sqrt{2}\int_0^{2\pi}\sqrt{2}\sin\frac{\theta}{2}\,d\theta$

$= a\,2\left[-\frac{2}{1}\cos\frac{\theta}{2}\right|_0^{2\pi}$

$= -4a(\cos\pi - \cos 0)$

$= -4a(-1-1) = 8a$

7. $A = 2\cdot\frac{1}{2}\int_0^{\pi} r^2\,d\theta$

$= \int_0^{\pi}(4+4\cos\theta+\cos^2\theta)\,d\theta$

$= 4\theta + 4\sin\theta\Big|_0^{\pi} + \int_0^{\pi}\dfrac{1+\cos 2\theta}{2}\,d\theta$

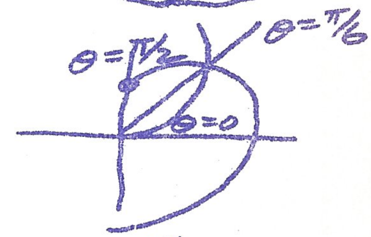
$= 4\pi + \left[\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi} = \dfrac{9\pi}{2}$

8. $r = 3\sin\theta$ $r = 2-\sin\theta$

$3\sin\theta = 2-\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6$



$A = 2\cdot\frac{1}{2}\int_0^{\pi/6}(3\sin\theta)^2\,d\theta + 2\cdot\frac{1}{2}\int_{\pi/6}^{\pi/2}(2-\sin\theta)^2\,d\theta$

$= 9\int_0^{\pi/6}\sin^2\theta\,d\theta + \int_{\pi/6}^{\pi/2}(4-4\sin\theta+\sin^2\theta)\,d\theta$

$= 9\int_0^{\pi/6}\dfrac{1-\cos 2\theta}{2}\,d\theta + 4\theta + 4\cos\theta\Big|_{\pi/6}^{\pi/2} + \int_{\pi/6}^{\pi/2}\dfrac{1-\cos 2\theta}{2}\,d\theta$

$= 9\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right|_0^{\pi/6} + 2\pi - \frac{2\pi}{3} - 2\sqrt{3} + \frac{1}{2}\theta - \frac{\sin 2\theta}{4}\Big|_{\pi/6}^{\pi/2}$

$= 9\left[\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right] + \frac{4\pi}{3} - 2\sqrt{3} + \frac{\pi}{4} - \frac{\pi}{12} + \frac{\sqrt{3}}{8} = \frac{9\pi}{4} - 3\sqrt{3}$