CALCULUS II EXAM 7B Dr. RAPALJE 15

Show all work on separate paper.

1. $\int \frac{dx}{x\sqrt{\ln x}}$

2. $\int \frac{\sqrt{1+e^{-2x}}}{e^{-3x}}\,dx$

3. $\int \frac{2x^2+1}{2x+1}\,dx$

4. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$

5. $\int_0^{\frac{\pi}{4}} \sin^2 x \cos^2 x\,dx$

6. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}\tan\sqrt{x}}\,dx$

7. $\int \frac{x^3\,dx}{(4+x^2)^{1/2}}$

8. $\int_2^{2\sqrt{3}} \frac{dx}{x^4\sqrt{16-x^2}}$

9. $\int \frac{dx}{(x^2+a^2)^2}$

CALCULUS II Exam 7B Solutions

Dr. RAPALJE

1. $\int \frac{dx}{x\sqrt{\ln x}}$ Let $u = \ln x$, $du = \frac{1}{x}dx$

$\int \frac{du}{u^{1/2}} = \int u^{-1/2}\,du = \frac{2}{1}u^{1/2} + c$

$= 2\sqrt{\ln x} + c$

2. $\int \frac{\sqrt{1+e^{-2x}}}{e^{-3x}}\,dx = \int \frac{\sqrt{1+\frac{1}{e^{2x}}}}{\frac{1}{e^{3x}}}\,dx$

$= \int \sqrt{\frac{e^{2x}+1}{e^{2x}}}\cdot\frac{e^{3x}}{1}\,dx = \int e^{2x}\sqrt{e^{2x}+1}\,dx$

$= \int u^{1/2}\frac{du}{2} = \frac{2u^{3/2}}{3\cdot 2} + c$. Let $u = e^{2x}+1$, $du = 2e^{2x}dx$

$= \frac{(e^{2x}+1)^{3/2}}{3} + c$

3. $\int \frac{2x^2+1}{2x+1}\,dx$

$$\begin{array}{r} x - \frac{1}{2} + \frac{3}{2}\cdot\frac{1}{2x+1} \\ 2x+1\,\overline{)\,2x^2 \qquad +1} \\ \underline{2x^2+x \qquad} \\ -x+1 \\ \underline{-x-\frac{1}{2}} \\ \frac{3}{2} \end{array}$$

$= \int x - \frac{1}{2} + \frac{3}{2}\frac{1}{2x+1}\,dx$

$= \frac{x^2}{2} - \frac{1}{2}x + \frac{3}{2}\frac{\ln|2x+1|}{2} + c$

$= \frac{x^2}{2} - \frac{1}{2}x + \frac{3}{4}\ln|2x+1| + c$

$\left(\frac{x^2}{2} - \frac{1}{2}x + \frac{3}{4}\ln|4x+2| + c \text{ also correct}\right)$

4. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$ Let $u = e^{\arctan x}$, $du = e^{\arctan x}\frac{1}{1+x^2}dx$

$\int du = u + c$

$= e^{\arctan x} + c$ or Let $u = \arctan x$, $du = \frac{1}{1+x^2}dx$

$\int e^u\,du = e^u + c$

$= e^{\arctan x} + c$.

5. $\int_0^{\pi/4} \sin^2 x\cos^2 x\,dx = \int_0^{\pi/4} \frac{1-\cos 2x}{2}\cdot\frac{1+\cos 2x}{2}\,dx$

$= \frac{1}{4}\int_0^{\pi/4} 1-\cos^2 2x\,dx = \frac{1}{4}\int_0^{\pi/4}\sin^2 2x\,dx$

$= \frac{1}{4}\int_0^{\pi/4}\frac{1-\cos 4x}{2}\,dx = \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/4} = \frac{\pi}{32}$

6. $\int \frac{\sec^2\sqrt{x}\,dx}{\sqrt{x}\tan\sqrt{x}}$ Let $u = \tan\sqrt{x}$, $du = \sec^2\sqrt{x}\,\frac{1}{2\sqrt{x}}dx$

$= 2\int\frac{du}{u} = 2\ln u + c$

$= 2\ln|\tan\sqrt{x}| + c$

7. $\int \frac{x^3\,dx}{(4+x^2)^{1/2}}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$\sqrt{4+x^2} = 2\sec\theta$

Let $u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$.

$= \int\frac{8\tan^3\theta\, 2\sec^2\theta\,d\theta}{2\sec\theta}$

$= 8\int\tan^3\theta\sec\theta\,d\theta$

$= 8\int\tan^2\theta(\tan\theta\sec\theta)$

$= 8\int(\sec^2\theta - 1)(\tan\theta\sec\theta\,d\theta)$

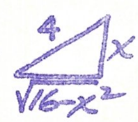
$= 8\int(u^2-1)\,du = 8\left[\frac{u^3}{3} - u\right] + c$

$= 8\left[\frac{\sec^3\theta}{3} - \sec\theta\right] + c$

$= 8\left[\frac{(\sqrt{4+x^2})^3}{3\cdot 8} - \frac{\sqrt{4+x^2}}{2}\right] + c$

$= \frac{1}{3}(4+x^2)^{3/2} - 4\sqrt{4+x^2} + c$

8. $\int_2^{2\sqrt{3}}\frac{dx}{x^4\sqrt{16-x^2}}$



x	$\cot\theta$
$2\sqrt{3}$	$\frac{1}{\sqrt{3}}$
2	$\sqrt{3}$

$= \int\frac{4\cos\theta\,d\theta}{4^4\sin^4 x\,4\cos\theta}$ $x = 4\sin\theta$, $dx = 4\cos\theta\,d\theta$

$= \frac{1}{256}\int\csc^4\theta\,d\theta$ $\sqrt{16-x^2} = 4\cos\theta$

$= \frac{1}{256}\int\csc^2\theta(\cot^2\theta+1)\,d\theta$ Let $u = \cot\theta$, $du = -\csc^2\theta\,d\theta$.

$= \frac{-1}{256}\int(u^2+1)\,du$

$= -\frac{1}{256}\left[\frac{u^3}{3}+u\right]_{\sqrt{3}}^{\frac{1}{\sqrt{3}}} = \frac{1}{256}\left[\frac{u^3}{3}+u\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$= \frac{1}{256}\left[\frac{3\sqrt{3}}{3} + \sqrt{3} - \frac{1\sqrt{3}}{3\cdot 3\sqrt{3}\sqrt{3}} - \frac{1\sqrt{3}}{\sqrt{3}\sqrt{3}}\right] = \frac{1}{256}\frac{(54-10)\sqrt{3}}{27} = \frac{11\sqrt{3}}{64\cdot 27}$

$\frac{11\sqrt{3}}{1728}$