CALCULUS III EXAM 5B

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1. Given the points $A(6,7,-2)$ $B(6,-5,-6)$

   a) Find the distance between A and B in simplest radical form.

   b) Give a set of direction numbers for $\overline{AB}$.

   c) Give a set of direction cosines for $\overline{AB}$.

   d) Give equation of line $\overline{AB}$ in parametric form.

   e) Give equation of line $\overline{AB}$ in non parametric form.

2. Find an equation of a line intersecting L_1: $\frac{x+5}{4} = \frac{y-1}{3} = \frac{z+8}{5}$ and perpendicular to it. (Hint: there are many such lines. Give one.)

3. Find the intersection of the line $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z-1}{-2}$ with the plane $3x - y + 2z - 3 = 0$.

4. Give equation of the plane passing through $(-1,2,4)$ which is perpendicular to L: $\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z}{1}$.

5. Find the intersection of the planes $3x + 2y - z + 5 = 0$
   (Use parametric form.) $2x + y + 2z - 3 = 0$

6. Find the equation of the plane through the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ which is parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

7. Express the rectangular coordinates $(-1, \sqrt{3}, -2)$

   a) in cylindrical coordinates.

   b) in spherical coordinates.

8. Give the most specific name for each surface in 3 dimensions:

   a) $y = x^2 + z^2$ d) $3x = 2y + 4z$

   b) $\frac{x^2}{9} - 1 = \frac{y^2}{4} + \frac{z^2}{9}$ e) $\frac{x^2}{4} = 4 - \frac{y^2}{4} - z^2$

   c) $4z - y^2 = 12$ f) $x = 4$

9. Graph 8a), 8c) 8e)

CALCULUS II Exam 5B Solutions Dr. RAPALJE

1. $A(6,7,-2)$ $B(6,-5,-6)$

a) $d=\sqrt{0^2+12^2+4^2}$
$=\sqrt{144+16}=\sqrt{160}=4\sqrt{10}$

b) dir. nos: $0, 12, 4$
or $0, 3, 1$

c) dir. cosines: $0, \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}$

d) $x=6+0t$ $\quad t=\frac{x-6}{0}$
$y=7+3t$ $\quad t=\frac{y-7}{3}$
$z=-2+t$ $\quad t=\frac{z+2}{1}$

e) $\frac{x-6}{0}=\frac{y-7}{3}=\frac{z+2}{1}$

or $x=6$; $\frac{y-7}{3}=\frac{z+2}{1}$

2. L_1: $\frac{x+5}{4}=\frac{y-1}{3}=\frac{z+8}{5}$

L: $\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$

i) Must have a point on the line L_1: $(x_0,y_0,z_0)=(-5,1,-8)$

ii) Must be ⊥ lines, so $4a+3b+5c=0$.

Let $a=3$, $b=-4$, $c=0$ or many other possibilities

$\frac{x+5}{3}=\frac{y-1}{-4}=\frac{z+8}{0}$

etc.

3. $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z-1}{-2}$

$x=3t-1$
$y=2t-1$
$z=-2t+1$

$3x-y+2z-3=0$
$3(3t-1)-(2t-1)+2(-2t+1)-3=0$
$9t-3-2t+1-4t+2-3=0$
$3t-3=0$
$t=1$
$x=2$, $y=1$, $z=-1$

4. Plane ⊥ $\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z}{1}$ has att nos. $4,-2,1$.
Passing through $(-1,2,4)$
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
$4(x+1)+-2(y-2)+1\cdot(z-4)=0$
$4x+4-2y+4+z-4=0$
$4x-2y+z+4=0$

5. $2(3x+2y-z+5=0)$
$2x+y+2z-3=0$

$6x+4y-2z+10=0$
$2x+y+2z-3=0$
$8x+5y\quad+7=0$
$8x=-5y-7$
Let $y=t$
$x=-\frac{5}{8}t-\frac{7}{8}$
$z=3x+2y+5$
$=-\frac{15}{8}t-\frac{21}{8}+2t+5$
$=\frac{1}{8}t+\frac{19}{8}$

$x=-\frac{5}{8}t-\frac{7}{8}$, $y=t$, $z=\frac{1}{8}t+\frac{19}{8}$ or $x=t$, $y=-\frac{8}{5}t-\frac{7}{5}$, $z=-\frac{1}{5}t+\frac{11}{5}$

or $-3x-2y+z-5=0$
$4x+2y+4z-6=0$
$x\quad+5z-11=0$
$x=-5z+11$ $\quad y=-2x-2z+3$
$\quad y=+10t-22-2t+3$
$\quad y=8t-19$

$x=-5t+11$
$y=8t-19$
$z=t$

(Dir Nos: $-5,8,1$)

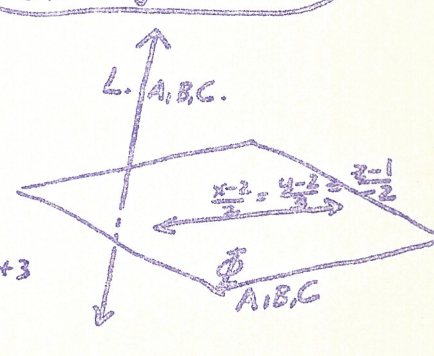
6. Plane through $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{-2}$

I $2A+2B+C+D=0$ $(2,2,1)$
II $2A+3B-2C\quad=0$ ⊥ lines.

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$

III $2A+3B+4C\quad=0$ ⊥ lines.

II−III. $C=0$
$2A=-3B$ Let $B=-2$
$A=3$
$D=-2A-2B-C$
$=-6+4$
$=-2$
$3x-2y-2=0$



7. $(-1,\sqrt{3},-2)$ Rectangular.
$r=\sqrt{1+3}=2$
$\tan\theta=-\sqrt{3}$ QII
$\theta=2\pi/3$ or $\cancel{5\pi/3}$

a) Cylindrical $\left(2, \frac{2\pi}{3}, -2\right)$

$\rho=\sqrt{1+3+4}=2\sqrt{2}$
$\cos\phi=\frac{z}{\rho}$ $\quad 0\le\phi\le\pi$
$=\frac{-2}{2\sqrt{2}}=-\frac{1}{\sqrt{2}}$
$\phi=\frac{3\pi}{4}$

b) Spherical $\left(2\sqrt{2}, \frac{2\pi}{3}, \frac{3\pi}{4}\right)$

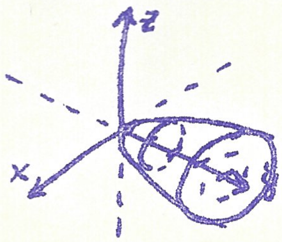
8a) $y = x^2 + z^2$

Circular paraboloid

xz trace $x^2+z^2=0$ point.

xy trace $y = x^2$ Parabola

yz trace $y = z^2$ Parabola



b) $\frac{x^2}{9} - \frac{y^2}{4} - \frac{z^2}{9} = 1$

Elliptic hyperboloid of 2 sheets.

xy trace: $\frac{x^2}{9} - \frac{y^2}{4} = 1$ Hyperbola.

xz trace: $\frac{x^2}{9} - \frac{z^2}{9} = 1$ Hyperbola

yz trace: $\frac{y^2}{4} + \frac{z^2}{9} = -1$ None

[yz sections $x \geq 3$ or $x \leq -3$ Ellipses]

c) $4z - y^2 = 12$

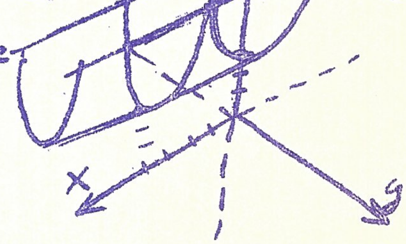
Parabolic cylinder.

z	y
3	0

yz trace: Parabola

xy trace: None

xz trace: $z=3$ line.



d) $3x = 2y + 4z$

Plane

[Traces are lines]

e) $\frac{x^2}{4} + \frac{y^2}{4} + z^2 = 4$

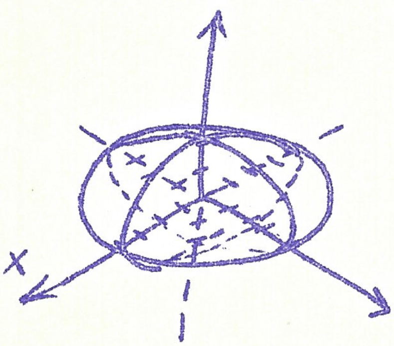
Oblate spheroid

$\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} = 1$

Traces: xy $\frac{x^2}{16} + \frac{y^2}{16} = 1$ Circle.

xz $\frac{x^2}{16} + \frac{z^2}{4} = 1$ Ellipse

yz $\frac{y^2}{16} + \frac{z^2}{4} = 1$ Ellipse.



f) $x = 4$

Plane $\perp$ x axis.