CALCULUS II EXAM 6B Vectors NAME Dr. RAPALJE

Show all work. Calculators, formula sheets allowed.

1. If $\vec{v} = 4\vec{i} + \vec{j}$, $\vec{w} = 6\vec{i} - 8\vec{j}$, find $|\vec{v}|$, $|\vec{w}|$, $\cos\theta$, and the projection of $\vec{v}$ on $\vec{w}$. (θ is angle between $\vec{v}$ and $\vec{w}$).

2. If $\vec{v} = 3\vec{i} - 4\vec{j}$ and $\vec{w} = 2\vec{i} + a\vec{j}$, determine the number a (if possible) so that
   a) $\vec{v}$ and $\vec{w}$ are parallel.
   b) $\vec{v}$ and $\vec{w}$ are orthogonal.

3. Find the work done by a force $F = 5\vec{i} - 2\vec{j} + 3\vec{k}$ when its point of application moves from $A(1,-2,2)$ to $B(3,1,-1)$.

4. Determine g and h so that $\vec{w} - g\vec{u} - h\vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$: $\vec{u} = 3\vec{i} - 2\vec{j}$; $\vec{v} = 2\vec{i} - \vec{k}$; $\vec{w} = 4\vec{i} - 2\vec{k}$.

5. If $\vec{u} = 4\vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{v} = -\vec{i} - 2\vec{j} + \vec{k}$, find $\vec{u} \times \vec{v}$.

6. $A(3,2,-2)$ $B(4,1,2)$, $C(1,2,3)$. Find the equation of the plane containing ABC, and the area of $\triangle ABC$.

7. Determine the volume of the parallepiped formed by $A(1,2,-3)$ $B(3,1,-2)$ $C(-1,3,1)$ $D(-3,4,3)$. If the points are coplanar, find the equation of the plane.

8. If $\vec{u} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{v} = \vec{i} + \vec{j} + 2\vec{k}$ $\vec{w} = 2\vec{i} - \vec{j} + 3\vec{k}$, find $(\vec{u} \times \vec{v}) \times \vec{w}$ and $\vec{u} \times (\vec{v} \times \vec{w})$. Are they the same?

9. If $\vec{f}(t) = e^{2t}\vec{i} + e^{-2t}\vec{j}$, find $\frac{d}{dt}\left[f'(t) \cdot f(t)\right]$

10. If $\vec{f}(t) = \cos\omega t\,\vec{i} + \sin\omega t\,\vec{j}$, where ω is a constant, find $\vec{v}(t)$, $|\vec{v}(t)|$, $\vec{a}(t)$, $|\vec{a}(t)|$, $s'(t)$, $s''(t)$.

1. $\vec V = 4\vec i + \vec j \quad \vec W = 6\vec i - 8\vec j$

$|\vec V| = \sqrt{4^2+1^2} = \sqrt{17}$

$|\vec W| = \sqrt{6^2+(-8)^2} = 10$

$\cos\theta = \dfrac{4\cdot 6 + 1\cdot(-8)}{\sqrt{17}\cdot 10} = \dfrac{8}{5\sqrt{17}}$

$\text{Proj}_{\vec W}\vec V = |\vec V|\cos\theta = \sqrt{17}\cdot\dfrac{8}{5\sqrt{17}} = \dfrac{8}{5}$

2a) $\vec V = 3\vec i - 4\vec j$

$\vec W = 2\vec i + a\vec j$

$\vec V \parallel \vec W$ if $\dfrac{3}{2} = \dfrac{-4}{a}$

$3a = -8$

$a = -8/3$

$\vec V \perp \vec W$ if $3\cdot 2 - 4\cdot a = 0$

$a = 3/2$

3. $\vec F = 5\vec i - 2\vec j + 3\vec k$

$A(1,-2,2) \quad B(3,1,-1)$

$\vec{AB} = 2\vec i + 3\vec j - 3\vec k$

$W = \vec F\cdot\vec{AB} = 10 - 6 - 9 = -5$

4. $(\vec w - g\vec u - h\vec v)\cdot\vec u = 0$

$[4\vec i - 2\vec k - g(3\vec i - 2\vec j) - h(2\vec i - \vec k)]\cdot(3\vec i - 2\vec j)$

$= [(4-3g-2h)\vec i + 2g\vec j + (-2+h)\vec k]\cdot(3\vec i - 2\vec j)$

$= 12 - 9g - 6h - 4g = 0; \quad 13g + 6h = 12 \quad \text{I}$

Also $(\vec w - g\vec u - h\vec v)\cdot\vec v = 0$

$= [(4-3g-2h)\vec i + 2g\vec j + (-2+h)\vec k]\cdot(2\vec i - \vec k)$

$= 8 - 6g - 4h + 2 - h = 0$

$5h + 6g = 10 \quad \text{II}.$

$5\,\text{I}: \quad 65g + 30h = 60$

$-6\,\text{II}: \quad -36g - 30h = -60$

$g = 0, \quad h = 2$

OR $\vec u\times\vec v = 2\vec i + 3\vec j + 4\vec k$

$\therefore 4 - 3g - 2h = 2c$

$2g = 3c$

$-2 + h = 4c$

$c = 0, \ g = 0, \ h = 2$

OR $(\vec w - g\vec u - h\vec v)\cdot\vec u = \vec w\cdot\vec u - g|\vec u|^2 - h\vec v\cdot\vec u = 0$

$12 - 13g - 6h = 0$

$(\vec w - g\vec u \cdot h\vec v)\cdot\vec v = \vec w\cdot\vec v - g\vec u\cdot\vec v - h|\vec v|^2 = 0$

$10 - 6g - 5h = 0$

5. $\vec u\times\vec v = \begin{vmatrix} \vec i & \vec j & \vec k \\ 4 & -2 & 3 \\ -1 & -2 & 1 \end{vmatrix} = \vec i\cdot 4 - \vec j\cdot 7 + \vec k\cdot(-10) = 4\vec i - 7\vec j - 10\vec k$

6. $A(3,2,-2) \quad B(4,1,2) \quad C(1,2,3)$

$\vec{AB} = \vec i - \vec j + 4\vec k$

$\vec{AC} = -2\vec i + 5\vec k$

$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & -1 & 4 \\ -2 & 0 & 5 \end{vmatrix}$

$\vec{AB}\times\vec{AC} = -5\vec i - 13\vec j - 2\vec k$ = vector $\perp$ plane.

Eq. plane: $-5(x-3) - 13(y-2) - 2(z+2) = 0$

$5x + 13y + 2z - 37 = 0$

Area $\Delta ABC = \frac{1}{2}|\vec{AB}\times\vec{AC}| = \frac{1}{2}|-5\vec i - 13\vec j - 2\vec k|$

$= \frac{1}{2}\sqrt{25+169+4} = \dfrac{\sqrt{198}}{2} = \dfrac{3\sqrt{22}}{2}$

7. $A(1,2,-3) \quad B(3,1,-2) \quad C(-1,3,1) \quad D(-3,4,3)$

$\vec{AB} = 2\vec i - \vec j + \vec k$

$\vec{AC} = -2\vec i + \vec j + 4\vec k$

$\vec{AD} = -4\vec i + 2\vec j + 6\vec k$

Volume $= (\vec{AB}\times\vec{AC})\cdot\vec{AD}$

$= \begin{vmatrix} 2 & -1 & 1 \\ -2 & 1 & 4 \\ -4 & 2 & 6 \end{vmatrix} = 0$

$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & -1 & 1 \\ -2 & 1 & 4 \end{vmatrix} = -5\vec i - 10\vec j$

($\perp$ plane)

Eq. of plane $-5(x-1) - 10(y-2) = 0$

$x - 1 + 2y - 4 =$

$x + 2y - 5 = 0$

8. $(\vec u\times\vec v)\times\vec w$

$= (\vec u\cdot\vec w)\vec v - (\vec v\cdot\vec w)\vec u$

$= 11(\vec i + \vec j + 2\vec k) - 7(3\vec i - 2\vec j + \vec k)$

$= -10\vec i + 25\vec j + 15\vec k$

$\vec u\times(\vec v\times\vec w)$

$= (\vec u\cdot\vec w)\vec v - (\vec u\cdot\vec v)\vec w$

$= 11(\vec i + \vec j + 2\vec k) - 3(2\vec i - \vec j + 3\vec k)$

$= 5\vec i + 14\vec j + 13\vec k$

Not same.

9. $\vec f(t) = e^{2t}\vec i + e^{-2t}\vec j$

$\vec f'(t) = 2e^{2t}\vec i - 2e^{-2t}\vec j$

$\vec f'(t)\cdot\vec f(t) = 2e^{4t} - 2e^{-4t}$

$\dfrac{d}{dt}[\vec f(t)\cdot\vec f(t)] = 8e^{4t} + 8e^{-4t}$

10. $\vec f(t) = \cos\omega t\,\vec i + \sin\omega t\,\vec j$

$\vec V(t) = -\omega\sin\omega t\,\vec i + \omega\cos\omega t\,\vec j$

$|\vec V(t)| = \sqrt{\omega^2\sin^2\omega t + \omega^2\cos^2\omega t}$

$= \omega = s'(t) = \text{const.}$

$\vec a(t) = -\omega^2\cos\omega t\,\vec i - \omega^2\sin\omega t\,\vec j$

$\vec a(t) = -\omega^2\vec f(t) \quad |\vec a(t)| = \omega^2$

$\vec f''(t) + \omega^2\vec f(t) = 0$, which is a differential equation.

$s''(t) = 0$