CALCULUS II EXAM 8B Dr. RAPALJE

1. $\int (x+2)\sqrt{x-1}\, dx$ (Give simplified, factored form.)

2. $\int 6x^2 \arcsin x \, dx$

3. $\int x^2 \cos 2x \, dx$

4. $\int \frac{x^2+3x+5}{x^3+8}\, dx$

5. $\int \frac{4x+7}{(x^2-2x+5)^2}\, dx$

Have a very Merry (and rest ful) Christmas!

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Dr. RAPALJE

1. $\int (x+2)\sqrt{x-1}\,dx$. Let $z=\sqrt{x-1}$, $z^2=x-1$, $2z\,dz=dx$, $x=z^2+1$

$= \int (z^2+3)\,z\cdot 2z\,dz$

$= 2\int (z^4+3z^2)\,dz$

$= 2\left[\frac{z^5}{5}+z^3\right]+c$

$= 2z^3\left(\frac{z^2+5}{5}\right)+c$

$= \frac{2}{5}(x-1)^{3/2}(x+4)+c$

2. $\int 6z^2 \arcsin x\,dx$ Let $u=\arcsin x$, $dv=6x^2dx$, $du=\frac{dx}{\sqrt{1-x^2}}$, $v=2x^3$

$= 2x^3\arcsin x - \int \frac{2x^3dx}{\sqrt{1-x^2}}$ Now use $z=\sqrt{1-x^2}$, $z^2=1-x^2$; $x^2=1-z^2$, $2z\,dz=-2x\,dx$, $z\,dz=-x\,dx$

$= 2x^3\arcsin x - 2\int \frac{(1-z^2)(-z\,dz)}{z}$

$= 2x^3\arcsin x + 2\int (1-z^2)\,dz$

$= 2x^3\arcsin x + 2z - \frac{2z^3}{3}+c$

$= 2x^3\arcsin x + 2\sqrt{1-x^2} - \frac{2}{3}(1-x^2)^{3/2}+c$

3. $\int x^2\cos 2x\,dx$ $u=x^2$, $dv=\cos 2x\,dx$, $du=2x$, $v=\frac{\sin 2x}{2}$

$= \frac{x^2}{2}\sin 2x - \int x\sin 2x\,dx$ $u=x$, $dv=\sin 2x\,dx$, $du=dx$, $v=\frac{-\cos 2x}{2}$

$= \frac{x^2}{2}\sin 2x - \left[-\frac{x\cos 2x}{2}+\int \frac{\cos 2x}{2}dx\right]$

$= \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + c$

4. $\frac{x^2+3x+5}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$

$x^2+3x+5 = A(x^2-2x+4)+(Bx+C)(x+2)$

let $x=-2$: $3=12A+0$ $\quad A=1/4$

$x=0$: $5=4A+2C$ $\quad C=2$

$x=1$: $9=3A+3B+3C$

$9=3/4+3B+6$ $\quad B=3/4$

$\int \frac{\frac{1}{4}dx}{x+2} + \int \frac{\frac{3}{4}x+2}{x^2-2x+4}dx$; $x^2-2x+4 = (x^2-2x+1)+3$; $y=x-1$, $dy=dx$

$= \frac{1}{4}\ln|x+2| + \int \frac{\frac{3}{4}(y+1)+2}{y^2+3}dy$

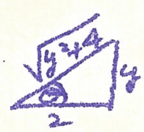
$= \frac{1}{4}\ln|x+2| + \frac{3}{4\cdot 2}\int\frac{2y\,dy}{y^2+3} + \frac{11}{4}\int\frac{dy}{y^2+3}$

$= \frac{1}{4}\ln|x+2| + \frac{3}{8}\ln|y^2+3| + \frac{11}{4\sqrt{3}}\arctan\frac{y}{\sqrt{3}}+c$

$= \frac{1}{4}\ln|x+2| + \frac{3}{8}\ln|x^2-2x+4| + \frac{11\sqrt{3}}{12}\arctan\frac{x-1}{\sqrt{3}}+c$

5. $\int \frac{4x+7}{(x^2-2x+5)^2}dx = \int \frac{[4(y+1)+7]}{(y^2+4)^2}dy$; $x^2-2x+5 = x^2-2x+1+4$; let $y=x-1$, $dy=dx$, $x=y+1$

$= \int \frac{4y+11}{(y^2+4)^2}dy$



$y=2\tan\theta$, $dy=2\sec^2\theta\,d\theta$, $\sqrt{y^2+4}=2\sec\theta$, $(y^2+4)^2=16\sec^4\theta$

$= \int\frac{4y\,dy}{(y^2+4)^2} + 11\int\frac{dy}{(y^2+4)^2}$

$\left[\text{let } u=y^2+4,\ du=2y\,dy\right]$ $\quad 11\int\frac{2\sec^2\theta\,d\theta}{16\sec^4\theta}$

$= \int\frac{2\,du}{u^2} + \frac{11}{8}\int\cos^2\theta\,d\theta$

$= 2\frac{u^{-1}}{-1} + \frac{11}{16}\int(1+\cos 2\theta)\,d\theta$

$= -2(y^2+4)^{-1} + \frac{11}{16}\left[\theta+\frac{1}{2}\sin 2\theta\right]+c$; $\frac{1}{2}\cancel{2}\sin\theta\cos\theta$

$= \frac{-2}{x^2-2x+5} + \frac{11}{16}\left[\arctan\frac{y}{2} + \frac{y}{\sqrt{y^2+4}}\cdot\frac{2}{\sqrt{y^2+4}}\right]+c = \frac{-2}{x^2-2x+5} + \frac{11}{16}\arctan\frac{x-1}{2} + \frac{11}{8}\cdot\frac{x-1}{x^2-2x+5}+c$

or $\frac{11x-27}{8(x^2-2x+5)} + \frac{11}{16}\arctan\frac{x-1}{2}+c$