CALCULUS II EXAM 3C 9.2 – 9.8 Dr. RAPALJE

Show all work on separate paper. Calculators, Blue sheet, Gold sheets, Chapter 9 formula sheet allowed.

1. Write an expression for the n^{th} term:

a) $2, -4, 6, -8, 10, \ldots$

b) $1, \frac{2}{4}, \frac{6}{9}, \frac{24}{16}, \ldots$

2. Find the sum of the geometric series:

a) $3 - 9/2 + 9/4 - 81/8 + \ldots$

b) $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}}$

3. Write $\sum_{n=2}^{\infty} \frac{4}{n(n+2)}$ as a telescoping series and find the sum.

4. A ball is dropped from a height of 100 ft. Each time it hits the ground it rebounds to half the original height. Express the distance traveled as an infinite summation, and find the total distance traveled.

In 5–10, determine convergence or divergence of the summations. Name each test used. For alternating series, show conditional or absolute convergence.

5. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ 6. $\sum_{n=1}^{\infty} e^{-n}$ 7. $\sum_{n=1}^{\infty} \frac{(-1)^n \cos n}{n\sqrt{n}}$ 8. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

9. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ 10. $\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^n$

In 11–12, find the interval of convergence. Check endpoints.

11. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n\, 5^n}$ 12. $\sum_{n=}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

In 13–14, approximate with error < 0.001:

13. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ 14. $\int_0^1 f(x)\,dx$ where $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$

15. Find interval of convergence for a) $f(x)$ b) $f'(x)$ c) $\int f(x)\,dx$ for $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2n}$
Test endpoints.

CALCULUS II EXAM 3C 9.2-9.8 Solutions Dr. RAPALJE

1a) $2, -4, 6, -8, 10, \ldots$

$(-1)^{n+1}(2n)$

b) $1, \frac{2}{4}, \frac{6}{9}, \frac{24}{16}, \ldots$

$\frac{n!}{n^2}$

2a) $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \ldots$

$r = -\frac{3}{2}$

$S = \frac{a}{1-r} = \frac{3}{1+\frac{3}{2}} = \frac{3}{\frac{5}{2}} = \frac{6}{5}$

b) $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}} = 3 + \frac{3}{2} + \frac{3}{4} + \ldots$

$S = \frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6$

3. $\sum_{n=2}^{\infty}\left[\frac{4}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}\right]$

$4 = A(n+2) + Bn$

$n=0 \quad 4 = 2A \quad A = 2$

$n=-2 \quad 4 = -2B \quad B = -2$

$2\sum_{n=2}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) =$

$= 2\left[\frac{1}{2} - \cancel{\frac{1}{4}} + \frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{4}} - \frac{1}{6} + \cancel{\frac{1}{5}} - \frac{1}{7} + \ldots\right]$

$= 2\left[\frac{1}{2} + \frac{1}{3}\right] = \frac{5}{3}$

4. $100 + 50 + 50 + 25 + 25 + \ldots$

$= 100 + 2[50 + 25 + \ldots]$

$= 100 + 2 \cdot \sum_{n=0}^{\infty} \frac{50}{2^n}$

$= 100 + 2 \cdot \frac{50}{1-\frac{1}{2}}$

$= 100 + 2 \cdot 100 = 300$ ft

5. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

Diverges by n^{th} term Thm.

6. $\sum_{n=1}^{\infty} e^{-n}$ Converges by Integral test

$\int_1^{\infty} e^{-x}dx = -e^{-x}\Big|_1^{\infty} = -\frac{1}{e}$

7. $\sum_{n=1}^{\infty} \frac{(-1)^n \cos n}{n^{3/2}} \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$

Converges Absolutely by comparison to p series.

8. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ Converges by Alt. Series Thm.

$\sum_{n=2}^{\infty} \frac{1}{\ln n} \geq \sum_{n=2}^{\infty} \frac{1}{n}$ Diverges

Comparison to harmonic series

Conditionally Convergent

9. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ Ratio test:

$\lim_{n\to\infty}\left|\frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n}\right|$

$= \lim_{n\to\infty} \frac{4}{n+1} = 0$

Converges

10. $\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^n$

Root test $\lim_{n\to\infty} \sqrt[n]{a^n}$

$= \lim_{n\to\infty} \frac{\ln n}{n} = \frac{\infty}{\infty}$

$= \frac{\frac{1}{n}}{1} = 0$

Converges

11. $\lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{(n+1)5^{n+1}} \cdot \frac{n5^n}{(x-4)^n}\right|$

$= \lim_{n\to\infty}\left|\frac{n}{n+1}\right|\left|\frac{x-4}{5}\right| < 1$

$|x-4| < 5$

$-5 < x-4 < 5$

$-1 < x < 9$

If $x = 9$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}5^n}{n5^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

Converges by alt series thm.

If $x=-1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5)^n}{n(5)^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n}{n}$

$= \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{n} = \sum_{n=1}^{\infty} \frac{-1}{n}$ Does not alt. Diverges Harmonic Series.

$-1 < x \leq 9$

12. $\lim_{n\to\infty}\left|\frac{x^{2n+2}}{(n+1)!} \cdot \frac{n!}{x^{2n}}\right| < 1$

$\lim_{n\to\infty}\left|\frac{x^2}{(n+1)}\right| < 1$

Converges for all x

$(-\infty, \infty)$

13. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ $|R_{n+1}| < .001$

$= 1 - \frac{1}{2} + \frac{1}{4 \cdot 2} - \frac{1}{8 \cdot 6} + \frac{1}{16 \cdot 24} - \boxed{\frac{1}{32 \cdot 120}}$ $(n+1)$

$\approx .607$

14. $\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 1^{2n+1}}{(2n+1)(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(2n+1)!}$

$= \frac{1}{1} - \frac{1}{3 \cdot 3!} + \frac{1}{5 \cdot 5!} - \frac{1}{7 \cdot 7!}$ $(n+1)$ $|a_{n+1}| < .001$

$= 1 - \frac{1}{18} + \frac{1}{600} \approx .94\bar{4} + .001\bar{6} \approx .946$

15a) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2n}$ $\quad 0 < x \leq 2$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^{n-1}}{2}$ $\quad 0 < x < 2$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{2n(n+1)}$ $\quad 0 \leq x \leq 2$

[Sorry, not room to show work]