

CALCULUS II PRACTICE QUIZZES AND EXAMS
QUIZ 1C Sections 5.8 - 6.5 Dr. R. Rapalje

Show all work on separate paper. Calculators allowed.

1. Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{\sqrt{x^2+4}}{x}\right)$

2. $\int \frac{(\ln x)^2}{x} dx$

3. Find $\frac{dy}{dx}$ for $y = e^{3x} \cdot \sin 2x$
(Give factored form)

4. Show that if $y = \arcsin x$, then $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
(Derive the formula as in class)

5. $\int \frac{2x-5}{x^2+4x+13} dx$

6. Find $f'(x)$ if $f(x) = x \arctan(2x) + \frac{1}{4} \ln(1+4x^2)$

CALCULUS II QUIZ | Sections 5.8 - 6.5 Solutions Dr. RAPALJE

1. $y = \ln \frac{\sqrt{x^2+4}}{x}$
 $= \frac{1}{2} \ln(x^2+4) - \ln x$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{x^2+4} \cdot 2x - \frac{1}{x}$$

$$= \frac{x}{x^2+4} - \frac{1}{x}$$

$$= \frac{x^2 - x^2 - 4}{x(x^2+4)} = \frac{-4}{x(x^2+4)}$$

2. $\int \frac{(\ln x)^2}{x} dx$ Let $u = \ln x$
 $du = \frac{1}{x} dx$

$$= \int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{1}{3} (\ln x)^3 + C$$

3. $y = e^{3x} \sin 2x$

$$\frac{dy}{dx} = e^{3x} (\cos 2x) \cdot 2 + (\sin 2x) e^{3x} \cdot 3$$

$$= e^{3x} (2 \cos 2x + 3 \sin 2x)$$

4. $y = \arcsin x$ $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$x = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\cos^2 y + \sin^2 y = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\cos y = \pm \sqrt{1 - \sin^2 y}$$

$$= \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$= \sqrt{1 - \sin^2 y}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

5. $\int \frac{2x-5}{x^2+4x+3} dx$

$$= \int \frac{2x-5}{x^2+4x+4+9} dx$$
 Let $u = x+2$
 $du = dx$

$$x = u-2$$

$$= \int \frac{2(u-2)-5}{u^2+9} dx$$

$$= \int \left(\frac{2u}{u^2+9} - \frac{9}{u^2+9} \right) du$$

$$= \ln(u^2+9) - 9 \cdot \frac{1}{3} \arctan \frac{x}{3} + C$$

$$= \ln(x^2+4x+13) - 3 \arctan \frac{x}{3} + C$$

6. $f(x) = x \arctan 2x - \frac{1}{4} \ln(1+4x^2)$

$$f'(x) = x \cdot \frac{1}{4x^2+1} \cdot 2 + (\arctan 2x) \cdot 1 - \frac{1}{4} \frac{1}{1+4x^2} \cdot 8x$$

$$= \frac{2x}{4x^2+1} + \arctan 2x - \frac{2x}{1+4x^2}$$

$$= \arctan 2x$$