CALCULUS II QUIZ 2 C Dr. RAPALJE

Volumes of revolution by shell and disk method.

1. Set up only to find the volume of revolution if the region bounded by $y = x^2$, $y = 0$, $x = 2$ is rotated about x axis:

   a) By disk method: b) By shell method:

2. The region in #1 is rotated about the y-axis:

   a) Set up for disk method b) Set up for shell method.

3. The region bounded by $y = x^2$, $x = 0$, $y = 4$ is rotated about the x axis:

   a) Set up for disk method b) Set up for shell method.

4. The region in #3 is rotated about the y axis:

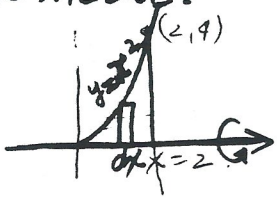
   a) Set up for disk method b) Set up for shell method.

CALCULUS II QUIZ 1C SOLUTIONS Dr. RAPALJE

Volumes of revolution by shell and disk method.

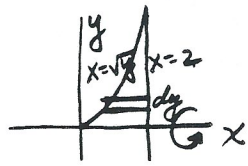
1. Set up only to find the volume of revolution if the region bounded by $y=x^2$, $y=0$, $x=2$ is rotated about x axis:

a) By disk method:



$$V=\pi\int_0^2 (x^2)^2\,dx$$
$$=\pi\int_0^2 x^4\,dx$$
$$=\pi\frac{x^5}{5}\Big|_0^2 = \frac{32\pi}{5}$$

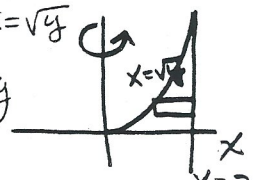
b) By shell method:



$$V=2\pi\int_0^4 y(2-\sqrt{y})\,dy$$
$$=2\pi\left[y^2-\frac{2y^{5/2}}{5}\right]_0^4 = 2\pi\left[16-\frac{64}{5}\right]$$
$$=2\pi\left(\frac{16}{5}\right)=\frac{32\pi}{5}$$

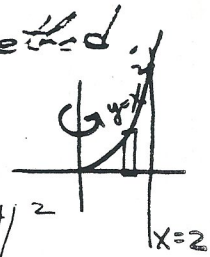
2. The region in #1 is rotated about the y-axis:

a) Set up for disk method



$$V=\pi\int 2^2-(\sqrt{y})^2\,dy$$
$$=\pi\int_0^4 (4-y)\,dy$$
$$=\pi\left[4y-\frac{y^2}{2}\right]_0^4=\pi(16-8)=8\pi$$

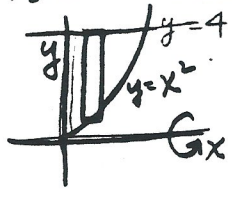
b) Set up for shell method.



$$V=2\pi\int x(x^2)\,dx$$
$$=2\pi\int_0^2 x^3\,dx = 2\pi\frac{x^4}{4}\Big|_0^2$$
$$=8\pi$$

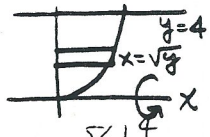
3. The region bounded by $y=x^2$, $x=0$, $y=4$ is rotated about the x axis:

a) Set up for disk method



$$\pi\int_0^2 4^2-(x^2)^2\,dx$$
$$\pi\int_0^2 (16-x^4)\,dx$$
$$\pi\,16x-\frac{x^5}{5}\Big|_0^2=\pi\left[32-\frac{32}{5}\right]=\frac{32\cdot 4\pi}{5}=\frac{128\pi}{5}$$

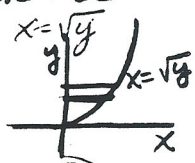
b) Set up for shell method.



$$2\pi\int_0^4 y\sqrt{y}\,dy$$
$$2\pi\int_0^4 y^{3/2}\,dy = 2\pi\frac{2y^{5/2}}{5}\Big|_0^4$$
$$=\frac{4\pi}{5}\cdot 2^5=\frac{128\pi}{5}$$

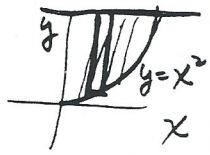
4. The region in #3 is rotated about the y axis:

a) Set up for disk method



$$\pi\int_0^4 (\sqrt{y})^2\,dy$$
$$\pi\int_0^4 y\,dy=\pi\frac{y^2}{2}\Big|_0^4=8\pi$$

b) Set up for shell method.



$$2\pi\int_0^2 x(4-x^2)\,dx$$
$$2\pi\left[\frac{4x^2}{2}-\frac{x^4}{4}\right]_0^2$$
$$=2\pi\left[8-4\right]=8\pi$$