CALCULUS II EXAM 2c (Chapter 8) Dr. RAPALJE

Show all work on separate paper.

1. $\int \frac{x^2}{x-1}\,dx$ 2. $\int \cos x\, e^{\sin x}\,dx$ 3. $\int \frac{e^x}{1+e^x}\,dx$

4. $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$ 5. $\int x^2 e^{2x}\,dx$ 6. $\int \arcsin x\,dx$

7. $\int \sec^4 3x \tan^3 3x\,dx$ 8. $\int \sin^2 x \cos^2 x\,dx$ 9. $\int \sin^5 x\,dx$

10. $\int \frac{dx}{\sqrt{x^2-6x+10}}$ 11. $\int \frac{x^2}{\sqrt{x^2-4}}\,dx$ 12. $\int_0^2 \sqrt{16-4x^2}\,dx$

13. $\int \frac{3x^2-7x-2}{x^3-x}\,dx$ 14. $\int \frac{x^2-1}{x^3+x}\,dx$

15. Set up only to find the value of $\int_0^4 x^3\,dx$, $n=4$
 a) Trapezoidal Rule
 b) Simpson's Rule

16. Find the maximum possible error for # 15a) and 15b).

17. $\lim_{x\to \pi/2} \frac{\sin x}{x-\pi/2}$ 18. $\lim_{x\to\infty} \left(1-\frac{2}{x}\right)^x$

19. $\int_{-1}^{2} \frac{dx}{x^2}$ 20. $\int_2^{\infty} \frac{dx}{x^2+4}$

Bonus Problem:

21. $\int \frac{1}{1+\sin x}\,dx$

CALCULUS II Exam 2C Solutions Dr. RAPALJE

1. $\int \frac{x^2}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right]dx$

$$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2} \\ \underline{x^2-x} \\ x \\ \underline{x-1} \\ 1 \end{array}$$

$= \frac{x^2}{2} + x + \ln|x-1| + c$

2. $\int \cos x\, e^{\sin x}\,dx$

$= \int e^u\,du$

$= e^u + c$

$= e^{\sin x} + c$

3. $\int \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$

$du = e^x dx$

$= \int \frac{du}{u}$

$= \ln u + c$

$= \ln(1+e^x) + c$

4. $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$ Let $u=\cos x$, $du = -\sin x\,dx$

$= \int u^{-1/2}(-du)$

$= -\frac{2u^{1/2}}{1} + c$

$= -2\sqrt{\cos x} + c$

5. $\int x^2 e^{2x}\,dx$ Let $u = x^2$, $dv = e^{2x}dx$, $du = 2x\,dx$, $v = \frac{e^{2x}}{2}$

$= \frac{1}{2}x^2e^{2x} - \int x e^{2x}\,dx$ Let $u = x$, $dv = e^{2x}dx$, $du = dx$, $v = \frac{e^{2x}}{2}$

$= \frac{1}{2}x^2e^{2x} - \left[\frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx\right]$

$= \frac{1}{2}e^{2x}\left(x^2 - x + \frac{1}{2}\right) + c$

6. $\int \arcsin x\,dx$ Let $u = \arcsin x$, $dv = dx$, $du = \frac{1}{\sqrt{1-x^2}}dx$, $v = x$

$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$w = \sqrt{1-x^2}$

$w^2 = 1-x^2$

$\cancel{2}w\,dw = -\cancel{2}x\,dx$

(Do you think they might sue me for using this?)

$= x\arcsin x + \int \frac{w\,dw}{w}$

$= x\arcsin x + w + c$

$= x\arcsin x + \sqrt{1-x^2} + c$

7. $\int \sec^4 3x\,\tan^3 3x\,dx$

Let $u = \tan 3x$, $du = 3\sec^2 3x\,dx$, $1+\tan^2 x = \sec^2 x$

$= \int (\sec^2 3x)\,u^3\,\frac{du}{3}$

$= \int (1+u^2)u^3\,\frac{du}{3}$

$= \frac{1}{3}\int (u^3+u^5)\,du$

$= \frac{1}{3}\left[\frac{u^4}{4} + \frac{u^6}{6}\right] + c = \frac{1}{12}\tan^4 3x + \frac{1}{18}\tan^6 3x + c$

7. Let $u = \sec 3x$

$du = 3\sec 3x\tan 3x\,dx$

$\int \sec^3 3x\tan^2 3x\,(\sec 3x\tan 3x\,dx)$

$= \int u^3(u^2-1)\frac{du}{3} = \frac{1}{3}\int (u^5 - u^3)\,du$

$= \frac{1}{3}\left[\frac{u^6}{6} - \frac{u^4}{4}\right] = \frac{1}{18}\sec^6 3x - \frac{1}{12}\sec^4 3x + c$

8. $\int \sin^2 x\cos^2 x\,dx$

$= \int \frac{1-\cos 2x}{2}\cdot\frac{1+\cos 2x}{2}\,dx$

$= \frac{1}{4}\int (1-\cos^2 2x)\,dx$

$= \frac{1}{4}\int \sin^2 2x\,dx$

$= \frac{1}{4}\int \frac{1-\cos 4x}{2}\,dx$

$= \frac{1}{8}x - \frac{\sin 4x}{32} + c$

9. $\int \sin^5 x\,dx$

$= \int \sin^4 x\,(\sin x\,dx)$

$u = \cos x$, $du = -\sin x\,dx$

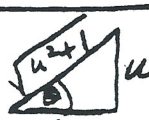
$= \int (1-\cos^2 x)^2(-du)$

$= -\int [1 - 2u^2 + u^4]\,du$

$= -u + \frac{2u^3}{3} - \frac{u^5}{5} + c$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$

10. $\int \frac{dx}{\sqrt{x^2-6x+9+1}}$



$u = x-3$, $du = dx$

$= \int \frac{dx}{\sqrt{(x-3)^2+1}}$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\sqrt{u^2+1} = \sec\theta$

$= \int \frac{du}{\sqrt{u^2+1}} = \int \frac{\sec^2\theta\,d\theta}{\sec\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + c$

$= \ln\left|\sqrt{u^2+1} + u\right| + c$

$= \ln\left|\sqrt{x^2-6x+10} + x - 3\right| + c$

11. $\int \frac{x^2\,dx}{\sqrt{x^2-4}}$

$= \int \frac{4\sec^2\theta(2\sec\theta\tan\theta\,d\theta)}{2\tan\theta}$ $\quad x = 2\sec\theta$, $dx = 2\sec\theta\tan\theta\,d\theta$, $\sqrt{x^2-4} = 2\tan\theta$

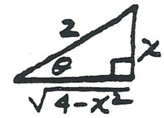
$= 4\int \sec^3\theta\,d\theta$

$= 4\cdot\frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right] + c$

$= 2\left[\frac{x}{2}\cdot\frac{\sqrt{x^2-4}}{2} + \ln\left|\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right|\right] + c$

$= 2\left[\frac{x\sqrt{x^2-4}}{4} + \ln\left|x+\sqrt{x^2-4}\right|\right] + c$

12. $\int_0^2 \sqrt{16-4x^2}\,dx$



$= 2\int_0^2 \sqrt{4-x^2}\,dx$ $\quad x = 2\sin\theta$, $dx = 2\cos\theta\,d\theta$, $\sqrt{4-x^2} = 2\cos\theta$

$= 2\int 2\cos\theta(2\cos\theta\,d\theta)$

$= 8\int \cos^2\theta\,d\theta$

$= 8\int \frac{1+\cos 2\theta}{2}\,d\theta$

$= 4\theta + \frac{4\sin 2\theta}{2}$

$= 4\theta + \frac{4\cancel{2}\sin\theta\cos\theta}{\cancel{2}}$

$= 4\arcsin\frac{x}{2} + 4\,\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}$

$= 4\arcsin 1 = 4\cdot\frac{\pi}{2} = 2\pi$

13. $\int \frac{3x^2-7x-2}{x(x^2-1)}\,dx = \int\left(\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}\right)dx$

$3x^2-7x-2 = A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1)$

if $x=0$ $\quad -2 = -A \quad A = 2$

$x=1 \quad -6 = 2B \quad B = -3$

$x=-1 \quad 8 = 2C \quad C = 4$

$= 2\ln x - 3\ln|x-1| + 4\ln|x+1| + c$

14. $\int \frac{x^2-1}{x(x^2+1)}\,dx = \int\left(\frac{A}{x} + \frac{Bx+C}{x^2+1}\right)dx$

$= A\int\frac{dx}{x} + \frac{B}{2}\int\frac{2x\,dx}{x^2+1} + C\int\frac{dx}{x^2+1}$

$= A\ln x + \frac{B}{2}\ln(x^2+1) + C\arctan x$

A = −1
B = 2
C = 0

15-16.

$f(x) = x^3 \quad x_0 = a = 0 \quad f(x_0) = 0$

$f'(x) = 3x^2 \quad x_1 = 1 \quad f(x_1) = 1$

$f''(x) = 6x \quad x_2 = 2 \quad f(x_2) = 8$

$f'''(x) = 6 \quad x_3 = 3 \quad f(x_3) = 27$

$f^{iv}(x) = 0 \quad x_4 = x_n = b = 4 \quad f(x_n) = 64$

15. a) $\frac{4-0}{8}\left[0 + 2\cdot1 + 2\cdot8 + 2\cdot27 + 64\right] = 68$

b) $\frac{4-0}{12}\left[0 + 4\cdot1 + 2\cdot8 + 4\cdot27 + 64\right] = 64$

16 a) Max $f''(x) = 6x$ is at $x=4$. So $E \le \frac{4^3}{12\cdot4^2}(24) = 8$

b) $f^{iv}(x) = 0$, so Simpson's Rule is exact. $E=0$

17. $\lim_{x\to\pi/2} \frac{\sin x}{x-\pi/2} = \frac{1}{0}$ DNE

Note: L'Hopital does not apply.

18. $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x$ Let $y = \left(1-\frac{2}{x}\right)^x$

$\ln y = x\ln\left(1-\frac{2}{x}\right) = \infty\cdot0$

$= \frac{\ln\left(1-\frac{2}{x}\right)}{1/x} = \frac{0}{0}$

$= \frac{\frac{1}{1-2/x}\cdot\frac{2}{x^2}}{-1/x^2} = -2 \quad \therefore y = e^{-2}$

19. $\int_{-1}^2 \frac{dx}{x^2} = \lim_{a\to0^-}\int_{-1}^a \frac{dx}{x^2} + \lim_{a\to0^+}\int_a^2 \frac{dx}{x^2}$

$= \lim_{a\to0^-} -x^{-1}\Big|_{-1}^a + \lim_{a\to0^+} -x^{-1}\Big|_a^2$ DNE. DIVERGES

20. $\lim_{a\to\infty}\int_2^a \frac{dx}{x^2+4} = \frac{1}{2}\arctan\frac{x}{2}\Big|_2^a$

$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$

21. $\int \frac{dx}{1+\sin x}\cdot\frac{(1-\sin x)}{(1-\sin x)}$

$= \int \frac{1-\sin x}{\cos^2 x}\,dx$

$= \tan x - \sec x + c$