CALCULUS II QUIZ 4C Dr. RAPALJE

Show all work. Justify answers.

1. Write an expression for the n^{th} term of the sequence:

a) $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \ldots$

b) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \ldots$

2. Find the sum of the sequence:

a) $2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \ldots,$

b) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

3. Determine convergence or divergence. Name each test.

a) $\sum_{n=1}^{\infty} \frac{1}{n+1}$

b) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

c) $\sum_{n=1}^{\infty} e^{-n}$

d) $\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)$

e) $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$

CALCULUS II QUIZ 4 Solutions

Dr. RAPALJE

Show all work. Justify answers.

1. Write an expression for the n^{th} term of the sequence:

a) $1, x, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \frac{x^5}{5!}, \ldots$ $\frac{x^{n-1}}{(n-1)!}$

b) $\frac{1}{2^1}, \frac{2}{2^2}, \frac{3}{2^3}, \frac{4}{2^4}, \frac{5}{2^5}, \ldots$ $\frac{2^n-1}{2^n}$

2. Find the sum of the sequence:

a) $2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \ldots$ $a=2$, $r=3/4$

$S = \frac{a}{1-r} = \frac{2}{1-3/4} = \frac{2}{1/4} = 8$

b) $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \left(\frac{1}{6} - \frac{1}{8}\right)$

$= \frac{3}{2}$

3. Determine convergence or divergence. Name each test.

a) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ Diverges — Integral Test; Limit Comparison Test $\sum \frac{1}{n}$

$\int_1^{\infty} \frac{dx}{x+1} = \ln(x+1)\Big|_1^{\infty} = \infty$ $\lim_{n\to\infty}\left|\frac{\frac{1}{n}}{\frac{1}{n+1}}\right| = \left|\frac{n+1}{n}\right| = 1$

b) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ Diverges — Integral test (Limit comparison test $\sum \frac{1}{n}$)

$\int_1^{\infty} \frac{x\,dx}{x^2+1} = \frac{1}{2}\ln(x^2+1)\Big|_1^{\infty} = \infty$

c) $\sum_{n=1}^{\infty} e^{-n}$ Converges — Integral test. Geometric Series $\sum (e^{-1})^n$ $r = e^{-1} < 1$

$\int e^{-x}\,dx = -e^{-x}\Big|_1^{\infty}$

$= -(0 - e^{-1})$

$= e^{-1}$

d) $\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)$ Diverges — $\lim_{n\to\infty} a_n \neq 0$. Divergence / n^{th} Term Thm

e) $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$ Converges — Limit Comparison test or Direct Comparison test. $\sum \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{3}{n(n+2)}}{\frac{1}{n^2}} = \frac{3n^2}{n^2+2n} = 3$

$\therefore$ Converges.