MS 229 EXAM 1

Determine whether or not each of the improper integrals is convergent and compute its value if it is:

Thought for Today: "I [Paul] came to realize that I could never find God's favor by trying – and failing – to obey laws. I came to realize that acceptance with God comes by believing in Christ."
Galatians 3: 19 Living Bible.

1. $\int_0^4 \frac{dx}{(x-1)^2}$

2. $\int_2^{+\infty} \frac{dx}{x(\ln x)^2}$

Find the length of the arc C:

3. $C=\{(x,y) : 2 \le x \le 4,\ y=\frac{1}{24}x^3+2x^{-1}\}$

Find the area of the surface S obtained by revolving the given arc C about the X axis:

4: $C=\{(x,y) : x=\frac{y^2}{4}-\frac{1}{2}\ln y \quad 1\le y\le 2\}$

Find the volume of revolution:

5. Use the disc method to find the volume of the region bounded by $y=x$ and $y=\sqrt{4x}$ revolved about the x axis.

6. Use the shell method to find the volume of the region R rotated about the y axis: $R=\{(x,y) : 1\le x\le 2,\ 0\le y\le \log x\}$

7. Use the disc method to find the volume of the region in #6 about the y axis:

8. Graph the region defined: $R=\{(x,y): 0\le x\le 1,\ x^2\le y\le \sqrt{x}\}$
(Shade the area)

9. Find the volume of the region bounded by the lines $x=\pi/3$, $y=0$, $y=\tan x$ rotated about the x axis.

E.C. Find the volume of the torus generated by revolving the circle $(x-3)^2+y^2=4$ about the y axis.

MS 229 Exam 1 Solutions

1. $\int_0^4 \frac{dx}{(x-1)^2} = \int_0^1 \frac{dx}{(x-1)^2} + \int_1^4 \frac{dx}{(x-1)^2}$

$= -(x-1)^{-1}\Big|_0^1 - (x-1)^{-1}\Big|_1^4$

$=$ $\boxed{\text{Divergent, undefined at } x=1.}$

2. $\int_2^\infty \frac{dx}{x(\ln x)^2}$  Let $u = \ln x$, $du = \frac{1}{x}dx$

$= \int u^{-2}\,du$

$= -u^{-1} = -\frac{1}{\ln x}\Big|_2^\infty = \lim_{a\to\infty} -\frac{1}{\ln x}\Big|_2^a$

$= \lim_{a\to\infty}\left(-\frac{1}{\ln a} + \frac{1}{\ln 2}\right) = \boxed{\frac{1}{\ln 2}}$

3. $\ell(C) = \int\sqrt{1+[y'(x)]^2}\,dx$  $y = \frac{1}{24}x^3 + 2x^{-1}$

$= \int\sqrt{1+\left(\frac{x^2}{8} - \frac{2}{x^2}\right)^2}\,dx$

$= \int\sqrt{1+\left(\frac{x^4-16}{8x^2}\right)^2}\,dx$

$= \int\sqrt{\frac{\cancel{64x^4}+x^8-8x^4+16^2}{64x^4}}\,dx$

$= \int\frac{x^4+16}{8x^2}\,dx = \int_2^4\left(\frac{1}{8}x^2 + 2x^{-2}\right)dx$

$= \frac{x^3}{24} - \frac{2}{x}\Big|_2^4 = \frac{64}{24} - \frac{1}{2} - \frac{8}{24} + 1$

$= \frac{56}{24} + \frac{1}{2} = \frac{68}{24} = \boxed{\frac{17}{6}}$

4. $A(S) = 2\pi\int y\,ds = 2\pi\int y\sqrt{1+[x'(y)]^2}\,dy$

$x = \frac{y^2}{4} - \frac{1}{2}\ln y$

$x' = \frac{y}{2} - \frac{1}{2y} = \frac{y^2-1}{2y}$

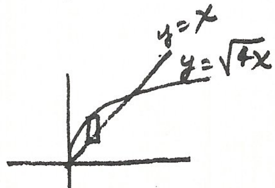
$A(S) = 2\pi\int_1^2 y\sqrt{1+\frac{y^4-2y+1}{4y^2}}\,dy$

$= 2\pi\int_1^2 y\,\frac{y^2+1}{2y}\,dy = \pi\int_1^2 y^2+1\,dy$

$= \pi\left[\frac{y^3}{3} + y\right]_1^2 = \frac{8}{3} + 2 - \frac{1}{3} - 1$

$= \pi\left[\frac{7}{3} + 1\right] = \boxed{\frac{10\pi}{3}}$

5. $y = x$  $y = \sqrt{4x}$




$x = \sqrt{4x}$ so $x^2 = 4x$,

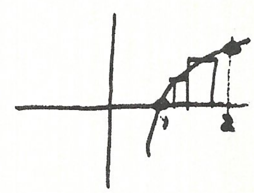
$x=0$ or $x=4$

$y=0$  $y=4$

$V = \pi\int_0^4\left[4x - x^2\right]dx = \pi\left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \pi\left[32 - \frac{64}{3}\right] = \pi\left(\frac{32}{3}\right) = \boxed{\frac{32\pi}{3}}$

6.




$V = 2\pi\int_1^2 xy\,dx$

$= 2\pi\int_1^2 x\ln x\,dx$  $u = \ln x$ $dv = x$, $du = \frac{dx}{x}$ $v = \frac{x^2}{2}$

$= 2\pi\left[\frac{x^2\ln x}{2} - \frac{x^2}{4}\right]_1^2$

$= 2\pi\left[\frac{4\ln 2}{2} - 1 - 0 + \frac{1}{4}\right]$

$= 2\pi\left[2\ln 2 - \frac{3}{4}\right] = \boxed{\frac{\pi}{2}(8\ln 2 - 3)}$

or $\boxed{4\pi\ln 2 - \frac{3\pi}{2}}$

7. $V = \pi\int_0^{\ln 2}\left(x_2^2 - x_1^2\right)dy$  $y = \log x$, $x = e^y$

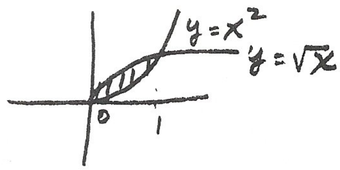
$= \pi\int_0^{\ln 2}\left(2^2 - e^{2y}\right)dy$

$= \pi\left[4y - \frac{e^{2y}}{2}\right]_0^{\ln 2} = \left(4\pi\ln 2 - \frac{\pi}{2}e^{2\ln 2} + \frac{\pi}{2}e^0\right)$

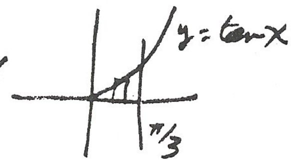
$= 4\pi\ln 2 - \frac{\pi}{2}e^{\ln 2^2} + \frac{\pi}{2}$

$= 4\pi\ln 2 - \frac{\pi}{2}\cdot 4 + \pi/2 = \boxed{4\pi\ln 2 - \frac{3\pi}{2}}$

8. $\{(x,y):\ 0 \le x \le 1,\ x^2 \le y \le \sqrt{x}\}$




9. *Disk method*




$$V = \pi \int_0^{\pi/3} \tan^2 x\, dx$$

$$= \pi \int_0^{\pi/3} (\sec^2 x - 1)\, dx$$

$$= \pi \Big[\tan x - x\Big]_0^{\pi/3} = \boxed{\pi\left[\sqrt{3} - \frac{\pi}{3}\right]}$$

E.C.