MS 229 EXAM 4C Dr. Rapalje

1. Determine if the differentials are exact, and if they are find the functions of which each is the differential.

a) $(x + y\cos x)\,dx + \sin x\,dy$

(7ea) b) $\dfrac{x\,dy - y\,dx}{x^2+y^2}$

c) $(2xy + z^2)\,dx + (2yz + x^2)\,dy + (2xz + y^2)\,dz$

2. Evaluate $\int_c (x+2y)\,dx + (x^2-y^2)\,dy$ where c is the arc of $y=x^3$ from $(0,0)$ to $(1,1)$. Is this path independent? Evaluate where
(10) c is the path from $(0,0)$ to $(1,0)$ and then $(1,0)$ to $(1,1)$.

3. Evaluate $\int_c (3x^2+6xy)\,dx + (3x^2-3y^2)\,dy$ over any arc C from
(10) $(1,1)$ to $(3,2)$. (Note: the differential is exact.)

4. Change the order of integration and integrate: $\int_0^2 \int_{2y}^4 e^{x^2}\,dx\,dy$
(10)

5. Evaluate: $\iint_F 4y\,dA$ where F is the region bounded by $y=x^2$ and $x+y=2$.
(10)

6. Set up the integral to find mass of the region F in two ways.
(10) Solve by either way: $\rho = 2x$; $F = \{(x,y):\ 0 \le x \le 1,\ x^3 \le y \le \sqrt{x}\}$

7. Evaluate by first changing to polar coordinates:

(10)
$$\int_0^4 \int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

8. Set up a triple integral and solve to find the volume of the solid
(10) bounded by $x^2+4y^2=4$, $z=0$, and $x+y+4z=4$.

9. Set up to find the area between $y=x^2$ and $x+y=2$ (See #5)
(10) using $\iint dx\,dy$ and $\iint dy\,dx$. Solve by easier means.

Thought for today: Therefore if any man (or woman presumably!) be in Christ, he is a new creature: old things are passed away; behold, all things are become new. II Corinthians 5:11.

MS 229 EXAM 4C Solutions Dr. Rapalje

1.a) $(x+y\cos x)\,dx + \sin x\,dy$

$\frac{\partial P}{\partial y} = \cos x \qquad \frac{\partial Q}{\partial x} = \cos x$

$\boxed{\text{Exact}}$. $\boxed{f(x,y) = \frac{x^2}{2} + y\sin x + c}$

b) $\frac{x\,dy - y\,dx}{x^2+y^2} = \frac{-y}{x^2+y^2}dx + \frac{x\,dy}{x^2+y^2}$

$\frac{\partial P}{\partial y} = \frac{(x^2+y^2)(-1) - (-y)(2y)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$ $\boxed{\text{Exact}}$

$\frac{\partial Q}{\partial x} = \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$f(x,y) = \int \frac{-y}{x^2+y^2}\partial x = -\arctan\frac{x}{y} + c_y$

$f(x,y) = \int \frac{x}{x^2+y^2}\partial y = \arctan y/x + c$ (same)

c) $(2xy+z^2)dx + (2yz+x^2)dy + (2xz+y^2)dz$

$\frac{\partial P}{\partial y} = 2x \qquad \frac{\partial Q}{\partial x} = 2x$

$\frac{\partial P}{\partial z} = 2z \qquad \frac{\partial R}{\partial x} = 2z \qquad \frac{\partial Q}{\partial z} = 2y \qquad \frac{\partial R}{\partial y} = 2y$ $\boxed{\text{EXACT}}$

$\boxed{f(x,y,z) = x^2y + xz^2 + y^2z + c}$

2. $\int (x+2y)\,dx + (x^2-y^2)\,dy \qquad y = x^3,\; dy = 3x^2dx,\; (0,0)$ to $(1,1)$

$= \int_0^1 (x+2x^3)dx + (x^2-x^6)(3x^2dx)$

$= \frac{x^2}{2} + \frac{x^4}{2} + \frac{3x^5}{5} - \frac{x^9}{3}\Big|_0^1 = 1 + \frac{3}{5} - \frac{1}{3}$

$= 1 + \frac{4}{15} = \boxed{\frac{19}{15}}$

Not path independent.

$\frac{\partial P}{\partial y} = 2 \qquad \frac{\partial Q}{\partial x} = 2x.$

over $(0,0)$ to $(1,0)$ and $(1,0)$ to $(1,1)$

$y=0,\ dy=0 \qquad x=1,\ dx=0$

$\int_0^1 x\,dx + \int_0^1 1-y^2\,dy = \frac{x^2}{2}\Big|_0^1 + \left(y-\frac{y^3}{3}\right)\Big|_0^1$

$= \frac{1}{2} + \frac{2}{3} = \boxed{\frac{7}{6}}$

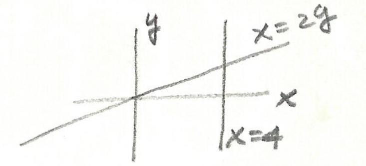
3. $\int (3x^2+6xy)dx + (3x^2-3y^2)dy \qquad (1,1)$ to $(3,2)$

$f(x,y) = x^3 + 3x^2y - y^3\Big|_{(1,1)}^{(3,2)}$

$= 27 + 54 - 8 - 3$

$= 81 - 11 = \boxed{70}$

4. $\int_0^2\int_{2y}^4 e^{x^2}\,dx\,dy$

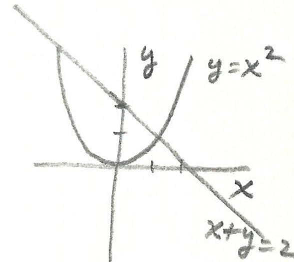


$\int_0^4\int_0^{x/2} e^{x^2}\,dy\,dx$

$= \int_0^4 e^{x^2}\frac{x}{2}\,dx \qquad u = x^2,\ du = 2x\,dx$

$= \frac{1}{4}e^{x^2}\Big|_0^4 = \boxed{\frac{1}{4}(e^{16}-1)}$

5. $\int_{-2}^1\int_{x^2}^{2-x} 4y\,dy\,dx$



$\int_{-2}^1 2y^2\Big|_{x^2}^{2-x}dx$

$= 2\int_{-2}^1 4-4x+x^2-x^4\,dx$

$= 2\left[4x - 2x^2 + \frac{x^3}{3} - \frac{x^5}{5}\right]_{-2}^1$

$= 2\left[4-2+\frac{1}{3}-\frac{1}{5} - \left(-8-8-\frac{8}{3}+\frac{32}{5}\right)\right]$

$= 2\left[2+\frac{1}{3}-\frac{1}{5}+16+\frac{8}{3}-\frac{32}{5}\right] = 2\left(21-\frac{33}{5}\right) = \boxed{\frac{154}{5}}$

$x + x^2 = 2$

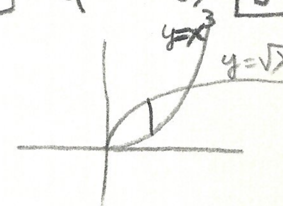
$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

$y = 4 \quad y = 1$

6. $\int_0^1\int_{x^3}^{\sqrt{x}} 2x\,dy\,dx$ or $\int_0^1\int_{y^2}^{\sqrt[3]{y}} 2x\,dx\,dy$



$= \int_0^1 x^2\Big|_{y^2}^{\sqrt[3]{y}}dy = \int_0^1 y^{2/3} - y^4\,dy$

$= \frac{3y^{5/3}}{5} - \frac{y^5}{5}\Big|_0^1 = \boxed{\frac{2}{5}}$

7. $\int_0^4\int_0^{\sqrt{4x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

$y = +\sqrt{4x-x^2}$

$y^2 = 4x - x^2$

$x^2 - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$

$x^2 + y^2 = 4x$

$r^2 = 4r\cos\theta$

$r = 4\cos\theta$

$= \int_0^{\pi/2}\int_0^{4\cos\theta} r\; r\,dr\,d\theta$

$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^{4\cos\theta}d\theta$

$= \frac{64}{3}\int_0^{\pi/2}\cos^3\theta\,d\theta$

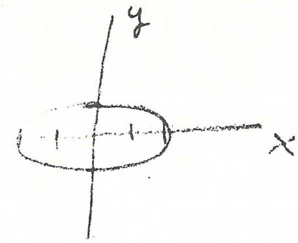
$= \frac{64}{3}\int (1-\sin^2\theta)\cos\theta\,d\theta$

$= \frac{64}{3}\int (1-u^2)\,du = \frac{64}{3}\left[u - \frac{u^3}{3}\right] \qquad u = \sin\theta,\ du = \cos\theta\,d\theta$

$= \frac{64}{3}\left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2} = \frac{64}{3}\cdot\frac{2}{3} = \boxed{\frac{128}{9}}$

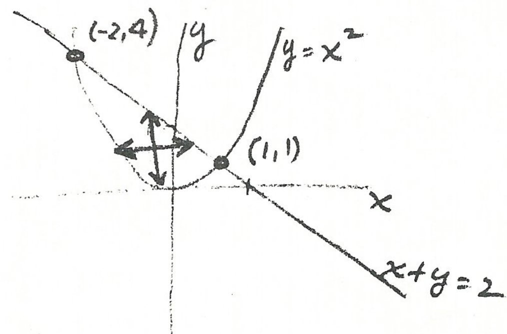
8. $$\int_{-1}^{1}\int_{-2\sqrt{1-y^2}}^{2\sqrt{1-y^2}}\int_{0}^{\frac{4-x-y}{4}} dz\,dx\,dy$$

$x = \pm\sqrt{4-4y^2}$



or $$\int_{-2}^{2}\int_{-\frac{\sqrt{4-x^2}}{2}}^{\frac{\sqrt{4-x^2}}{2}}\int_{0}^{\frac{4-x-y}{4}} dz\,dy\,dx$$

9. $$\int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy + \int_{1}^{4}\int_{-\sqrt{y}}^{2-y} dx\,dy$$



or $$\int_{-2}^{1}\int_{x^2}^{2-x} dy\,dx = \int_{-2}^{1}(2-x-x^2)\,dx$$

$$= 2x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_{-2}^{1}$$

$$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4-2+\frac{8}{3}\right)$$

$$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$$

$$= 5 - \frac{1}{2} = \boxed{9/2}$$