DIFFERENTIAL EQUATIONS EXAM 3 C Solutions

Dr. Rapalje

1a) $\mathcal{L}\{t^3e^{-2t}\}$ $F(t)=t^3$

$\mathcal{L}\{e^{-2t}F(t)\}=\dfrac{3!}{(s+2)^4}$

b) $\mathcal{L}\{t\cdot t\sin\omega t\}$ $F(t)=t\sin\omega t$

$\mathcal{L}\{t\,F(t)\}=(-1)F'(s)$

$=(-1)\dfrac{d}{ds}\left(\dfrac{2\omega s}{(s^2+\omega^2)^2}\right)$

$=\dfrac{2\omega(3s^2-\omega^2)}{(s^2+\omega^2)^3}$

c) $\mathcal{L}\{\sin^2 t\}$

$=\mathcal{L}\left\{\dfrac{1-\cos 2t}{2}\right\}$

$=\dfrac{1}{2}\cdot\dfrac{1}{s}-\dfrac{1}{2}\cdot\dfrac{s}{s^2+4}$

$=\dfrac{2}{s(s^2+4)}$

2a) $\mathcal{L}^{-1}\left\{\dfrac{10s}{(s+3)^2+16}\right\}=\mathcal{L}^{-1}\left\{\dfrac{10(s+3)-30}{(s+3)^2+16}\right\}$

$=10\mathcal{L}^{-1}\left\{\dfrac{s+3}{(s+3)^2+16}\right\}-\dfrac{30}{4}\mathcal{L}^{-1}\left\{\dfrac{4}{(s+3)^2+16}\right\}$

$=10e^{-3t}\cos 4t-\dfrac{15}{2}e^{-3t}\sin 4t$

b) $\mathcal{L}^{-1}\left\{\dfrac{s^2-6}{s(s+3)(s+1)}\right\}=\mathcal{L}^{-1}\left\{\dfrac{A}{s}+\dfrac{B}{s+3}+\dfrac{C}{s+1}\right\}$

$s^2-6=A(s+3)(s+1)+Bs(s+1)+Cs(s+3)$

$A=-2,\quad B=\frac{1}{2}\quad C=\frac{5}{2}$

$\mathcal{L}^{-1}=-2+\frac{1}{2}e^{-3t}+\frac{5}{2}e^{-t}$

c) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}=\mathcal{L}^{-1}\left\{\dfrac{A}{s+1}+\dfrac{B}{(s+1)^2}\right\}$

$s=A(s+1)+B$

$A=1\quad B=-1$

$\mathcal{L}^{-1}=e^{-t}-te^{-t}$

d) $\mathcal{L}^{-1}\left\{\dfrac{5s^2-7s+17}{(s-1)(s^2+4)}\right\}=\mathcal{L}^{-1}\left\{\dfrac{A}{s-1}+\dfrac{Bs+C}{s^2+4}\right\}$

$5s^2-7s+17=A(s^2+4)+(Bs+C)(s-1)$

$A=3\quad B=2\quad C=-5$

$\mathcal{L}^{-1}\left\{\dfrac{3}{s-1}\right\}+\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2+4}\right\}-\dfrac{5}{2}\mathcal{L}^{-1}\left\{\dfrac{2}{s^2+4}\right\}$

$=3e^{t}+2\cos 2t-\frac{5}{2}\sin 2t$

3. $y''+4y=\sin 2t\qquad y(0)=y'(0)=2$

$s^2y-sY(0)-Y'(0)+4y=\dfrac{2}{s^2+4}$

$(s^2+4)y=\dfrac{2}{s^2+4}+2s+2$

$y=\dfrac{2}{(s^2+4)^2}$ * $+\dfrac{2s}{s^2+4}+\dfrac{2}{s^2+4}$

$Y(t)=\cancel{2}\cdot\dfrac{1}{\cancel{2}\cdot 2^3}(\sin 2t-2t\cos 2t)$
$+2\cos 2t+\sin 2t$

$=\frac{1}{8}\sin 2t-\frac{1}{4}t\cos 2t+2\cos 2t+\sin 2t$

$=\frac{9}{8}\sin 2t+2\cos 2t-\frac{1}{4}t\cos 2t$

4. $y''+4y=\csc 2x$

$y_c=c_1\sin 2x+c_2\cos 2x$

$y_{Gen}=u_1\sin 2x+u_2\cos 2x$

$y'=2u_1\cos 2x+u_1'\sin 2x-2u_2\sin 2x+u_2'\cos 2x$

Impose: $u_1'\sin 2x+u_2'\cos 2x=0$

$y''=-4u_1\sin 2x+2u_1'\cos 2x-4u_2\cos 2x-2u_2'\sin 2x$

$-\sin 2x\,(2u_1'\cos 2x-2u_2'\sin 2x=\csc 2x)$

$2\cos 2x\,(u_1'\sin 2x+u_2'\cos 2x=0)$

$2u_2'\sin^2 2x+2u_2'\cos^2 2x=-1$

$u_2'=-\frac{1}{2}\qquad u_2=-\frac{1}{2}x+c_2$

$u_1'=\frac{1}{2}\cot 2x\qquad u_1=\frac{1}{4}\ln|\sin 2x|+c_1$

$y=\left(\frac{1}{4}\ln|\sin 2x|+c_1\right)\sin 2x+\left(-\frac{1}{2}x+c_2\right)\cos 2x$

$=c_1\sin 2x+c_2\cos 2x$
$+\frac{1}{4}\ln|\sin 2x|\sin 2x-\frac{1}{2}x\cos 2x$

* Given $\mathcal{L}^{-1}\left\{\dfrac{1}{(s^2+a^2)^2}\right\}=\dfrac{1}{2a^3}(\sin at-at\cos at)$