MAC 2233 EXAM 4D **Dr. Rapalje** **NAME** ______________________

Chapter 5

Show all work on separate paper. Turn in ALL worksheets.

1. $\int (24x^2 - 8x + 12 + x^{-3})\,dx$

2. $\int \left(6\sqrt{x} - 5 + \frac{6}{\sqrt[3]{x}}\right) dx$ 3. $\int \left(e^{3x} - \frac{1}{e^{3x}}\right) dx$

4. $\int \left(\frac{1}{x^2} + \frac{1}{x} + 1\right) dx$ 5. $\int \frac{3x^3 - 5x^2 + 6x - 7}{x}\,dx$

6. Given $\int_0^2 (x^3 + 4e^{-2x})\,dx$
   a) Find the exact value using calculus.
   b) Find the decimal approximation (using the calculator!)

7. Find the area under the curve $f(x) = 3x^2 - 4x + 5$ from $x = 2$ to $x = 5$. Set up the integral and solve using calculus. Check with the calculator.

8. Find the area between the curves $y = 12x - 3x^2$ and $y = 6x - 24$.

9. Find the average value of the function $f(x) = e^{\frac{1}{2}x}$ on [0, 2]. Give the exact value.

In 10 – 13, find each integral.

10. $\int (x^3 + 5)^5 x^2 dx$ 11. $\int e^{x^2} x dx$ 12. $\int \frac{x^3 dx}{x^4 + 4}$ 13. $\int \frac{(\ln x)^3}{x} dx$

14. Evaluate: $\int_0^3 x\sqrt{x^2 + 9}\,dx$ a) using calculus (exact value)
    b) using calculator (decimal approx)
    c) $\int_0^4 x\sqrt{x^2 + 9}\,dx$ either method.

15. World consumption of aluminum is running at the **rate** of $72\,e^{0.06t}$ million tons per year where t is the number of years since year 2000. Find a formula for the total amount of aluminum consumed within t years of 2000. If this rate continues, how long will it take to exhaust the known resources of 8500 million tons?

MAC 2233 EXAM 4D Solutions Dr. Rapalje

1. $\int (24x^2 - 8x + 12 + x^{-3})\,dx$

$= 8x^3 - 4x^2 + 12x + \frac{x^{-2}}{-2} + C$

$= 8x^3 - 4x^2 + 12x - \frac{1}{2x^2} + C$

2. $\int (6\sqrt{x} - 5 + \frac{6}{\sqrt[3]{x}})\,dx$

$= \int (6x^{1/2} - 5 + 6x^{-1/3})\,dx$

$= 6\cdot\frac{2}{3}\cdot x^{3/2} - 5x + 6\cdot\frac{3}{2}x^{2/3} + C$

$= 4x^{3/2} - 5x + 9x^{2/3} + C$

3. $\int (e^{3x} - \frac{1}{e^{3x}})\,dx$

$= \int (e^{3x} - e^{-3x})\,dx$

$= \frac{e^{3x}}{3} - \frac{e^{-3x}}{-3} + C$

$= \frac{1}{3}(e^{3x} + e^{-3x}) + C$

4. $\int (\frac{1}{x^2} + \frac{1}{x} + 1)\,dx$

$= \int (x^{-2} + x^{-1} + 1)\,dx$

$= \frac{x^{-1}}{-1} + \ln x + x + C$

$= -\frac{1}{x} + \ln x + x + C$

5. $\int \frac{3x^3 - 5x^2 + 6x - 7}{x}\,dx$

$= \int (3x^2 - 5x + 6 - \frac{7}{x})\,dx$

$= x^3 - \frac{5}{2}x^2 + 6x - 7\ln x + C$

6. a) $\int_0^2 (x^3 + 4e^{-2x})\,dx$

$= \frac{x^4}{4} + \frac{4e^{-2x}}{-2}\Big|_0^2$

$= (\frac{16}{4} - 2e^{-4}) - (0 - 2e$

$= 4 - 2e^{-4} + 2$

$= 6 - 2e^{-4}$ EXACT VALUE

b) ≈ 5.96 (2 ways to calculate!)

7. $\int_2^5 (3x^2 - 4x + 5)\,dx$

$= x^3 - 2x^2 + 5x\Big|_2^5$

$= (5^3 - 2(5^2) + 5\cdot 5) - (2^3 - 2\cdot 2^2 + 5\cdot 2)$

$= (125 - 50 + 25) - (8 - 8 + 10)$

$= 100 - 10 = 90$

8. Upper Lower

$12x - 3x^2 = 6x - 24$

$0 = 3x^2 - 6x - 24$

$0 = 3(x^2 - 2x - 8)$

$0 = 3(x-4)(x+2)$

$x = 4 \quad x = -2$

$\int_{-2}^{4} [(12x - 3x^2) - (6x - 24)]\,dx$

$\int_{-2}^{4} (-3x^2 + 6x + 24)\,dx$

$= -x^3 + 3x^2 + 24x\Big|_{-2}^{4}$

$= -(4^3) + 3(4^2) + 24(4)$

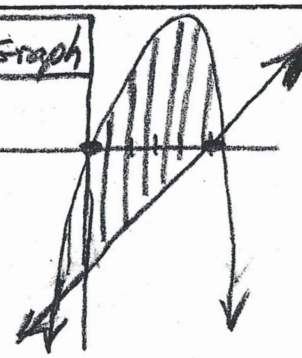
$\quad - (-(-2)^3 + 3(-2)^2 + 24(-2))$

$= -64 + 48 + 96$

$\quad - (8 + 12 - 48)$

$= 80 - (-28) = 108$

8.Graph



9. $Av = \frac{1}{b-a}\int f(x)\,dx$

$= \frac{1}{2-0}\int_0^2 e^{1/2 x}\,dx$

$= \frac{1}{2}\cdot\frac{2}{1}e^{1/2 x}\Big|_0^2$

$= e^1 - e^0 = e - 1$ EXACT!

(≈ 1.72)

10. $\int (x^3 + 5)^5 x^2\,dx$ Let $u = x^3 + 5$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \int u^5 \frac{du}{3}$

$= \frac{1}{3}\cdot\frac{u^6}{6} + C$

$= \frac{1}{18}(x^3 + 5)^6 + C$

11. $\int e^{x^2} x\,dx$ Let $u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int e^u \frac{du}{2}$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{x^2} + C$

12. $\int \frac{x^3 dx}{x^4+4}$ Let $u = x^4+4$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$= \int \frac{\frac{du}{4}}{u}$

$= \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln u + c$

$= \frac{1}{4}\ln(x^4+4) + c$

13. $\int \frac{(\ln x)^3}{x} dx$ Let $u = \ln x$

$du = \frac{1}{x} dx$

$= \int u^3\, du$

$= \frac{u^4}{4} + c = \frac{(\ln x)^4}{4} + c$

14a) $\int_0^3 x\sqrt{x^2+9}\, dx$ Let $u = x^2+9$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int \sqrt{x^2+9} \cdot x\,dx$

$= \int u^{1/2} \frac{du}{2}$

$= \frac{1}{2} \frac{2u^{3/2}}{3}$

$= \frac{1}{3}(x^2+9)^{3/2} \Big|_0^3$

$= \frac{1}{3}\left(18^{3/2} - 9^{3/2}\right)$

$= \frac{1}{3}\left(18\sqrt{18} - 27\right)$

$= \frac{1}{3}\left(18 \cdot 3\sqrt{2} - 27\right)$

$= 18\sqrt{2} - 9$

EXACT VALUE

b) Calculator (in 2 ways!)

≈ 16.46

c) $\int_0^4 x\sqrt{x^2+9}\, dx$

= (Same integration!)

$= \frac{1}{3}(x^2+9)^{3/2}\Big|_0^4$

$= \frac{1}{3}\left(25^{3/2} - 9^{3/2}\right) = \frac{1}{3}(125-27) = \frac{1}{3}(98) = 98/3$ EXACT VALUE

$\approx 32.\overline{6}$

Let y = # million tons consumed

15. Rate $= \frac{dy}{dt}$

$\frac{dy}{dt} = 72e^{0.06t}$

$y = \int 72e^{0.06t} dt$

$y = \frac{72}{0.06} e^{0.06t} + c$

$y = 1200e^{0.06t} + c$

At $\cancel{t}=0$ in year 2000,

y = # million tons consumed $= 0$

$0 = 1200e^0 + c$

$0 = 1200 + c$

$c = -1200$

$y = 1200e^{0.06t} - 1200.$

When will $y = 8500$ (all gone!)?

$8500 = 1200e^{0.06t} - 1200$

$\frac{9700}{1200} = \frac{1200e^{0.06t}}{1200}$

$\frac{97}{12} = e^{0.06t}$

$\ln\left(\frac{97}{12}\right) = \ln e^{0.06t}$

$\frac{\ln\left(\frac{97}{12}\right)}{0.06} = \frac{0.06t}{0.06}$

$t = \frac{\ln\left(\frac{97}{12}\right)}{.06}$

$t \approx 34.83$ years